Introduction to particle accelerators II. Beam dynamics

Nicolas Delerue
LAL (CNRS) and Université de Paris-Sud

Reminder

- To produce electrons they must be extracted from a cathode. This can be done by heating it or by shooting at it with a laser.
- Ions must be extracted from a plasma.
- To reach high energies RF acceleration is required.
- Magnets are used to keep the particles in the right orbit.

However

- We ignored:
  - That particles have the same charge and they repel each other
  - Some particles may have the wrong phase…
  - If they are packed too close from each other they may hit each other
  - Charged particles undergoing an acceleration should emit radiations…

Lectures overview

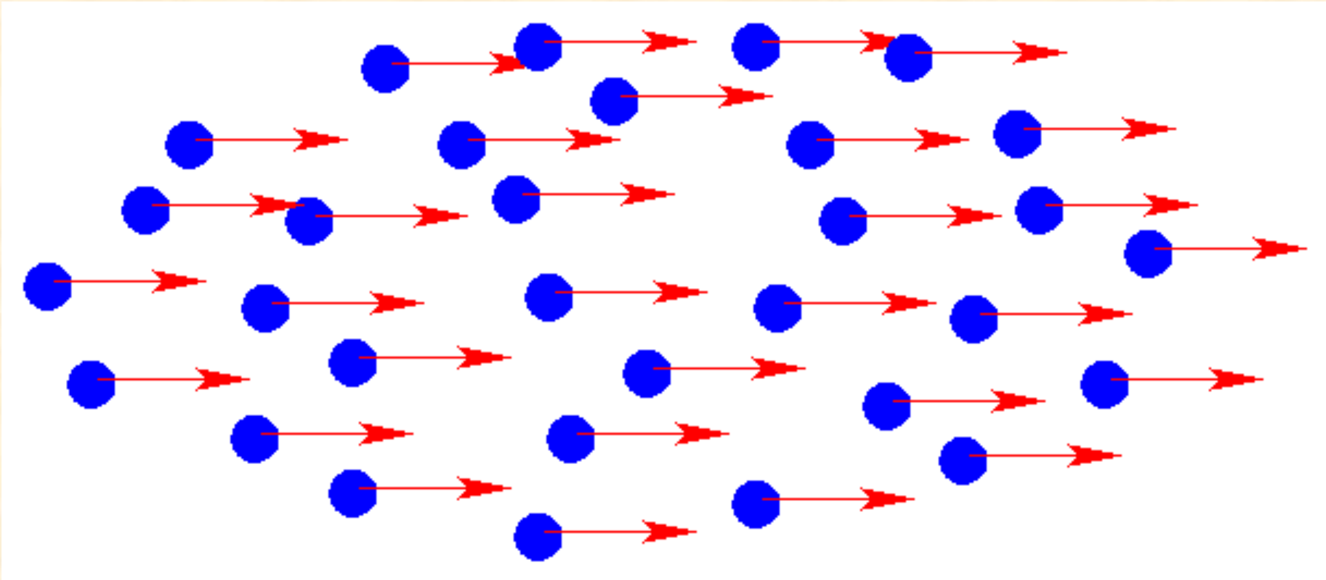
I. Basic principles and overview

II. Beam dynamics

- Emittance
- Impedance issues
- Tune
- Intrabeam scattering
- Synchrotron radiation
- Longitudinal dynamics
- Beam beam effects

III. Diagnostics and applications

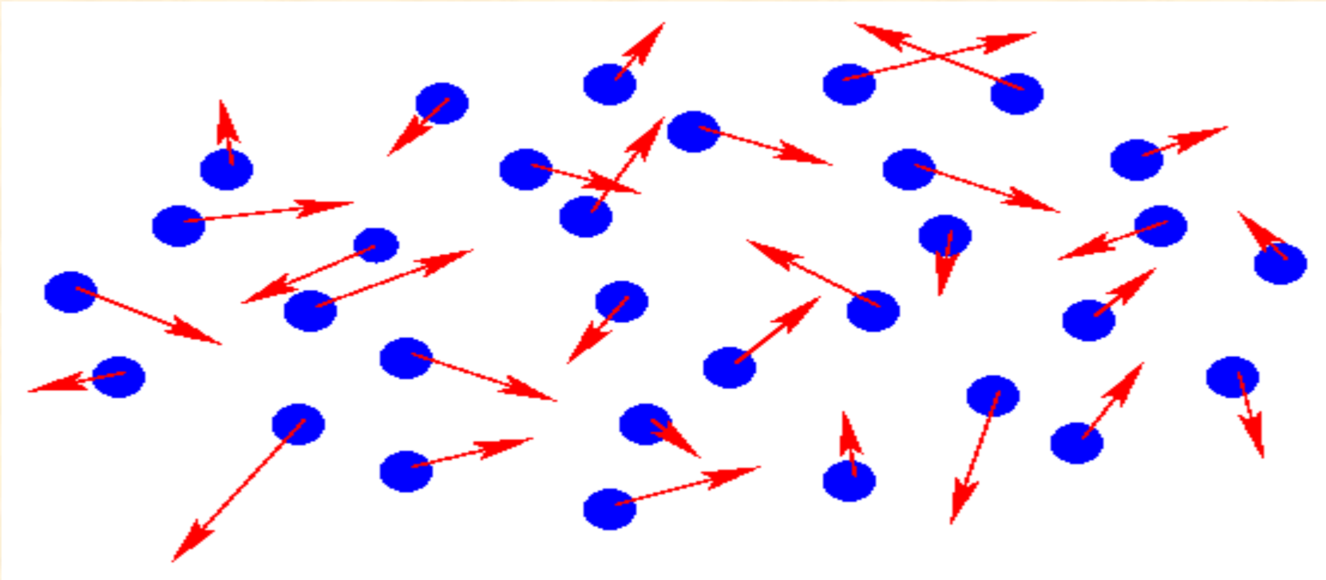
Let's look at a particle bunch



An observer in the laboratory frame looking at a particle bunch will only see particles travelling at the speed of light, apparently all in the same direction.

It is very different if one looks in the bunch's centre of mass frame...

Let's look at a particle bunch



In the centre of mass of the bunch, the particles do not look so well organised...

This should remind you other statistical systems that you have already studied: Gases!

Perfect gas law

- You have studied earlier that a perfect gas obeys the law: PV=nRT
- V is the volume term (V=xyz)
- P is a dynamic term: P, the pressure is proportional to the amount of scattering experienced by atoms as they travel in the volume. It is proportional to the momentum of the gas atoms (P~x'y'z').
- Hence it is possible to write that for gas atoms the product of their position by their momentum is expressed by their temperature *(times a constant).*
- We have seen that in the CoM particles look like a perfect gas. The product of their position by their momentum is called the “emittance” of the beam.

6D Trace space

- The position-momentum 6D space is called the trace space.
- To help visualisation the trace space can be decomposed in 3 orthogonal position-momentum planes:

$$xyzx'\ y'\ z' = xx'*\ yy'*\ zz'$$

- It is also often useful to look separately at the transverse and longitudinal planes.

Liouville's theorem

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \sum_{i=1}^{d}\left(\frac{\partial\rho}{\partial q^i}\dot{q}^i + \frac{\partial\rho}{\partial p_i}\dot{p}_i\right) = 0.$$

- The volume occupied in the phase space by a system of particles is constant.
- This is a general physics theorem, not limited to accelerators.
- The application of external forces or the emission of radiation needs to be treated carefully.

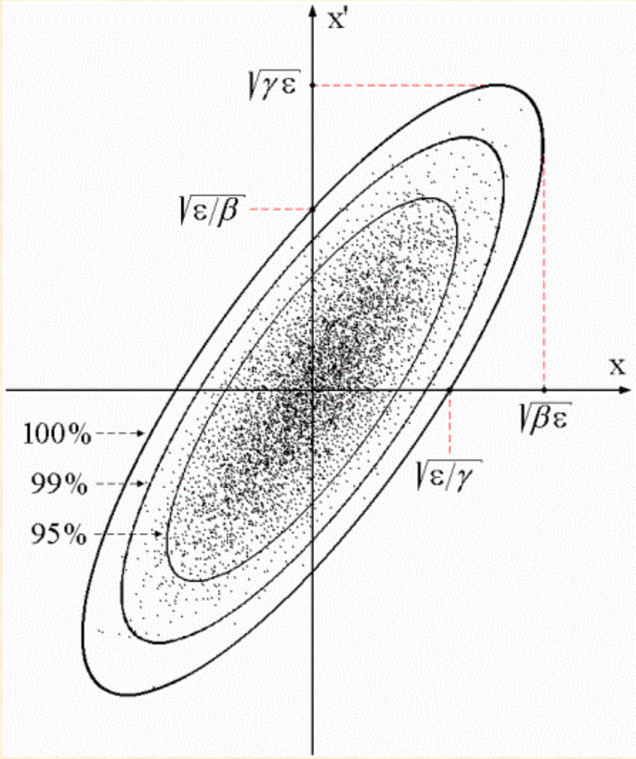
Joseph Liouville
1809-1882
(source: wikipedia)

Emittance

- We have defined the emittance as the volume occupied by the beam in the trace space.
- Liouville's theorem tells us that such volume must be constant.
- Hence the emittance of a beam is constant (unless external forces are applied).

- The total volume occupied by the bunch in trace-space is usually dominated by a few far-outlying particles.
- Instead of giving the volume occupied by all the particles, it is common to give the volume occupied by 90% or 60% of the particles or to give the RMS emittance.
- The fraction of particles included in the emittance is usually quoted.

$\in_{90}$ $\in_{RMS}$



Quizz

1) Is the emittance of a particle beam an intensive or an extensive physical property?

- a) Intensive
- b) Extensive

2) In which units should the transverse emittance in 1 dimension (x,x') be expressed?

- a) Square metres
- b) Barns
- c) Metre x radians
- d) Kelvin

Answer: (1b)

- Emittance is the product of the beam size by the beam divergence.
- Both are extensive quantities
- The emittance of a fraction of a beam will be smaller than the emittance of the full beam.

$$\in_{90} > \in_{60}$$

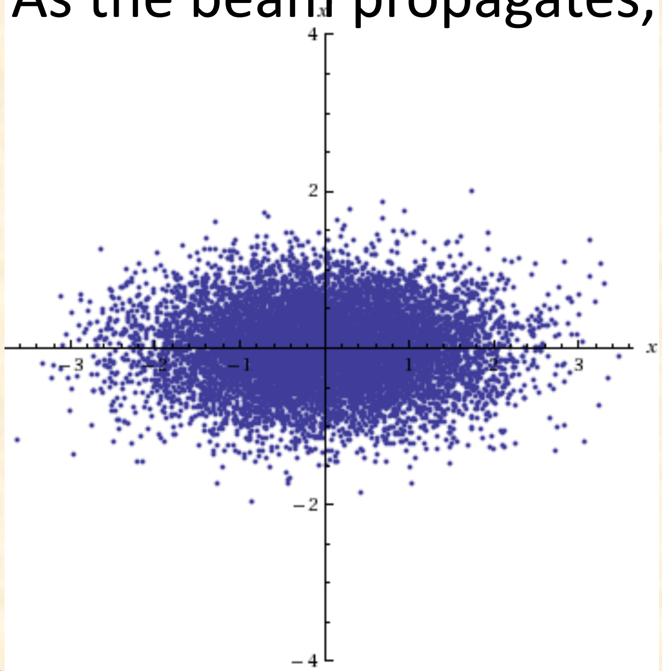
- This property is used in particle accelerators: the emittance of a beam can be improved by trimming it.

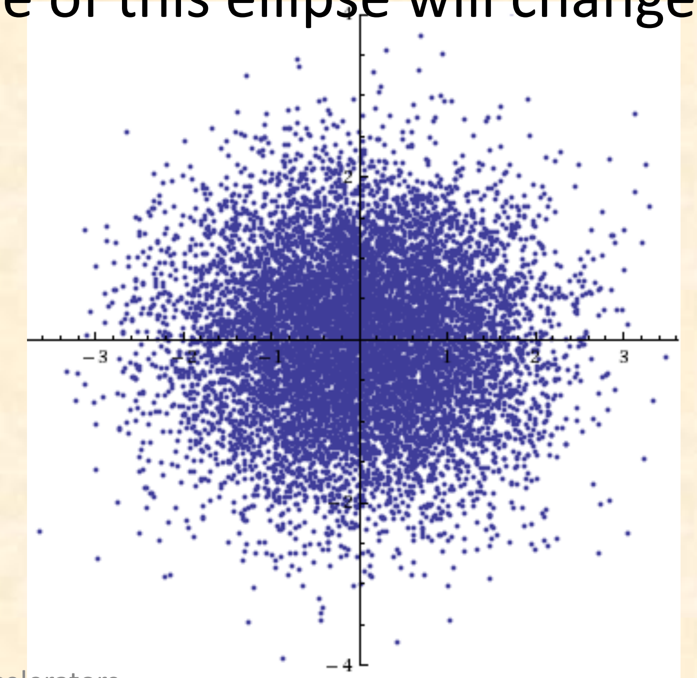
Answer: (2c)

- The size of the beam is expressed in (milli)metres.
- The divergence of the beam can be expressed either in (milli) radians or in eV.
- Depending on conventions a factor Pi may be added.
- A typical emittance for an electron linac with a thermionic gun is mm.mrad.
- A synchrotron can go below nm.mrad.
- Do not forget that 1 mm.mrad = 1um.rad!

Emittance ellipse

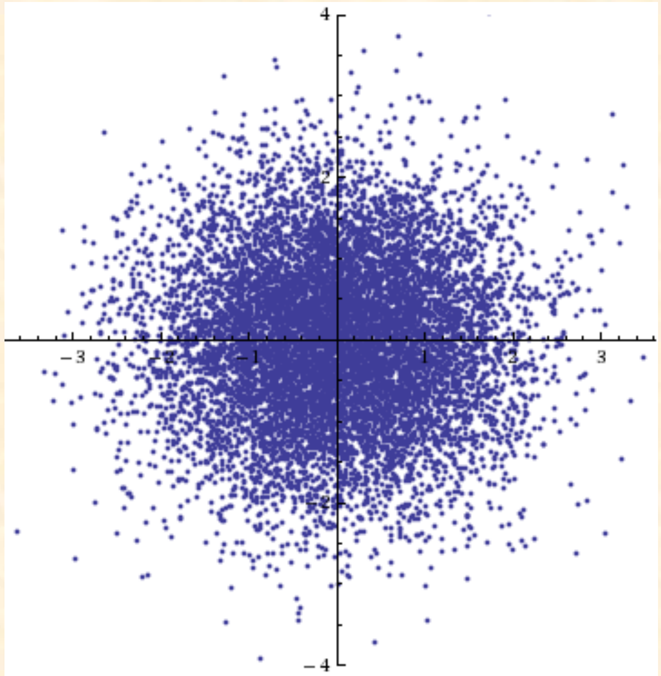
- A random gaussian distribution of particles forms a straight ellipse.
- By choosing the right set of coordinates this ellipse can be transformed in a circle (do not forget that the two axis are orthogonal!).
- As the beam propagates, the shape of this ellipse will change.





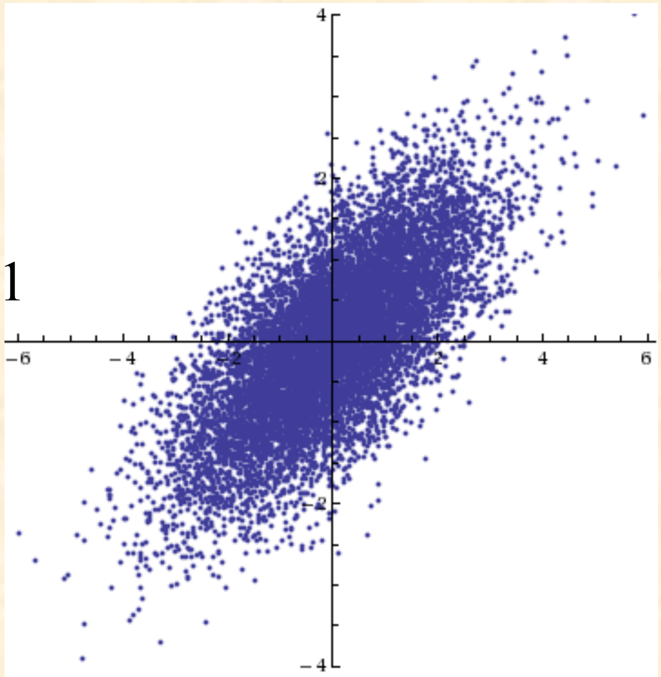
Beam drift

- When the beam "drifts" that is, propagates in space, over a length L with no external forces applied:
  - The momentum of the particles is constant
  - The position changes by the momentum times L.
- Hence, the emittance ellipse is sheared.



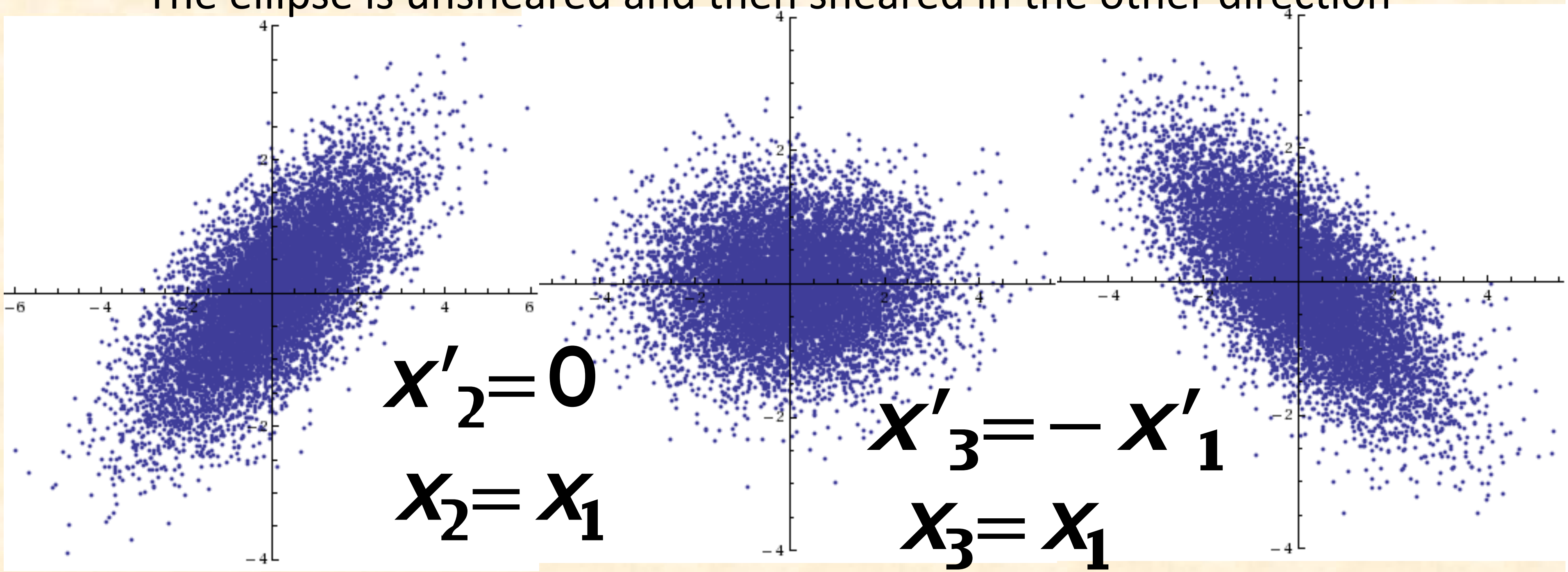
$$x'_2 = x'_1$$

$$x_2 = x_1 + Lx'_1$$



Focussing

- In a focussing section (typically a quadrupole), in the thin lens approximation:
  - The position of the particles is not affected
  - The momentums are reversed, hence a waist (at which all x'=0) is formed.
- The ellipse is unsheared and then sheared in the other direction

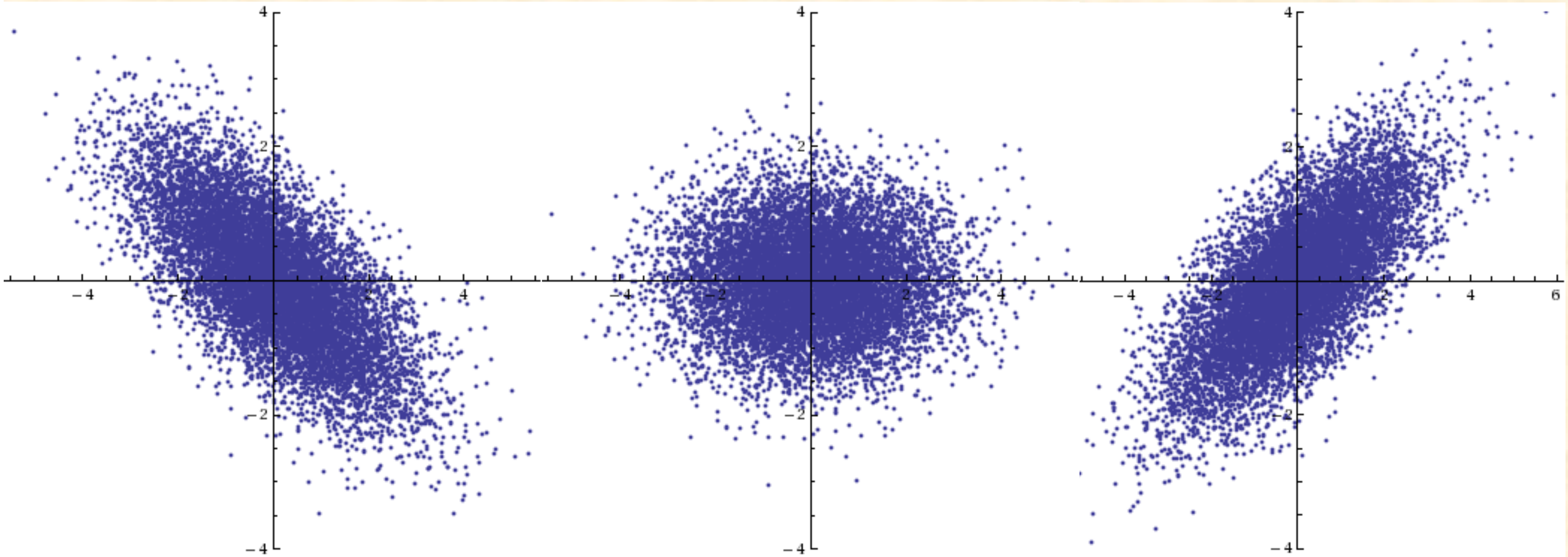


$$x'_2 = 0$$
$$x_2 = x_1$$

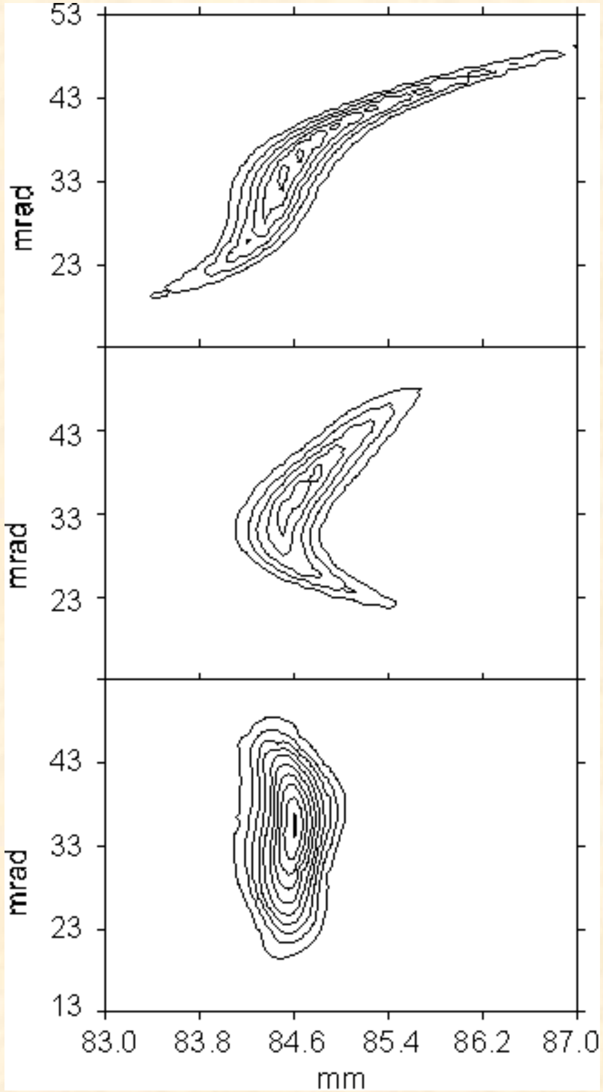
$$x'_3 = -x'_1$$
$$x_3 = x_1$$

Beam waist

- After the focussing section the beam will drift again, decreasing the shearing of the emittance ellipse.
- At some point the momentums will again average to 0, the beam will be forming a waist.
- At the waist the shearing of the emittance ellipse flips and starts increasing again.
- The beam size is the smallest at the waist.



Non linear effects



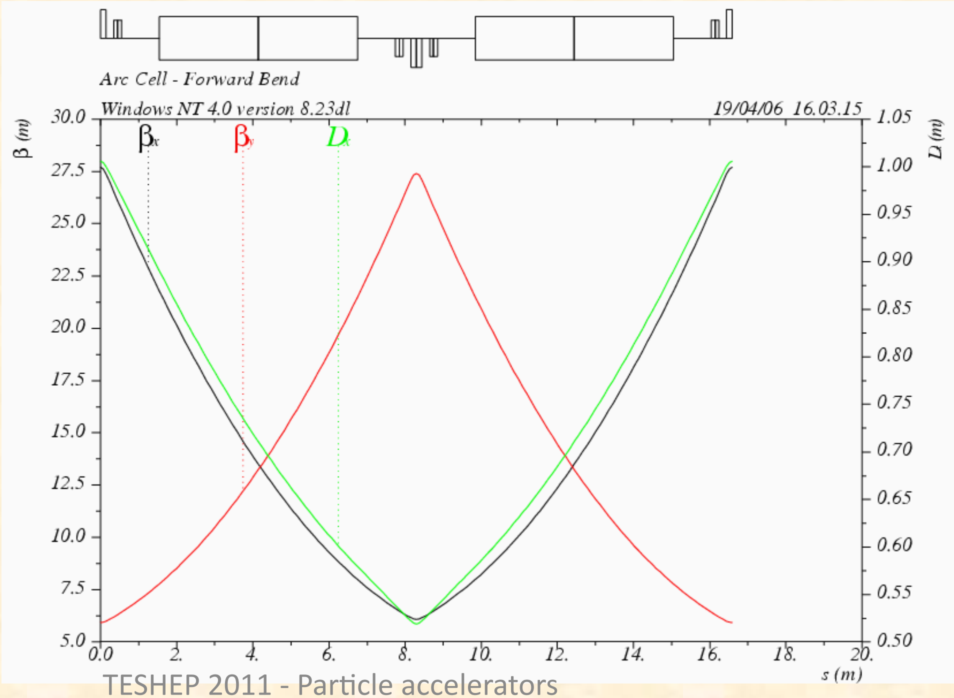
Source:TRIUMF

- Magnet non linearities will increase the deformation of the emittance ellipse.
- Higher order magnets are required to correct such deformations (octupole and sextupole).
- Example of emittance measured at an accelerator in Canada.

Beta function

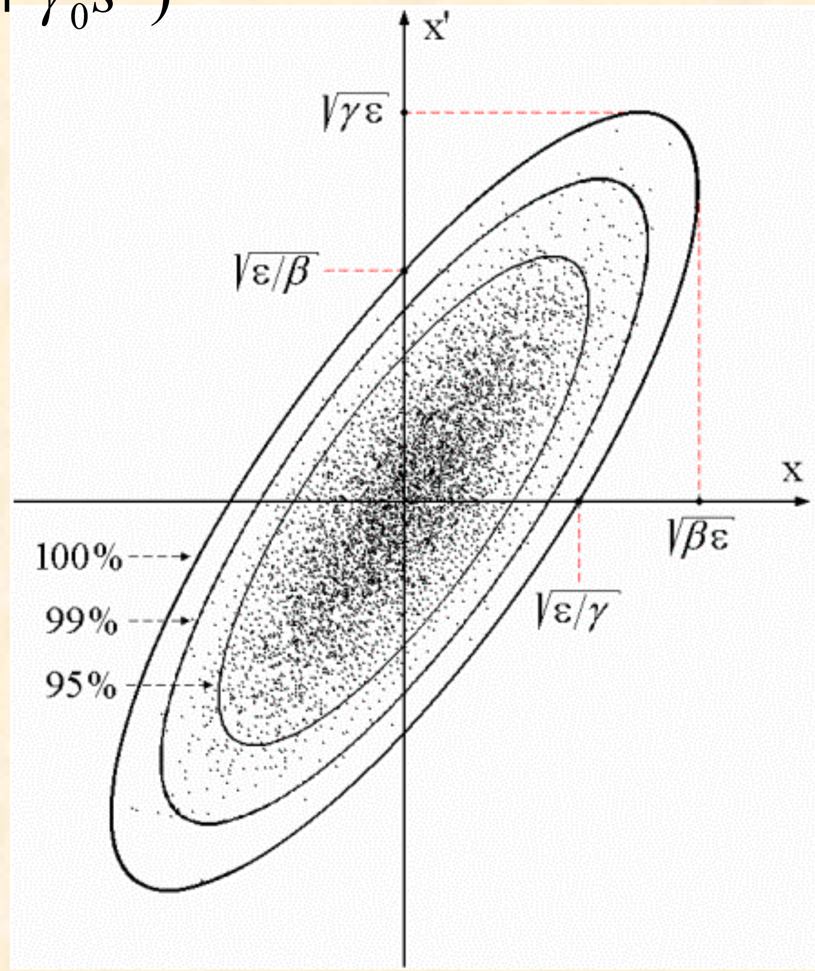
It is convenient to define the “beta function” to relate the beam size to the emittance.

$$\beta = \frac{\sigma^2}{\in} \qquad \sigma = \sqrt{\beta \in} \qquad \sigma_{RMS} = \sqrt{\beta \in_{RMS}}$$



Beam parametrisation: Twiss parameters

$$\sigma = \sqrt{\in(\beta_0 - 2\alpha_0 s + \gamma_0 s^2)}$$

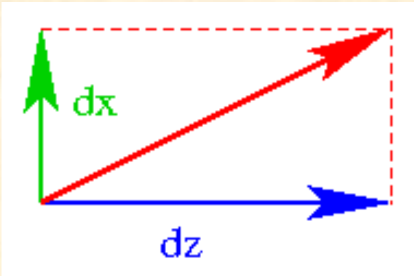


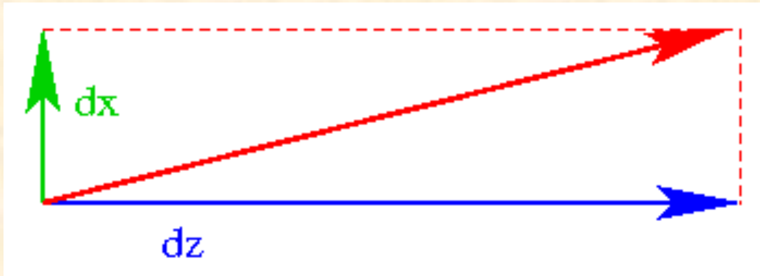
- The emittance ellipse can be described using the Courant-Snyder representation (Twiss parameters).
- Like any other physical system in which elements travels in straight line, the beam envelope forms an hyperbola.

$$\sigma = \sqrt{\in(\beta_0 + \frac{s_w^2}{\beta_0})}$$

Acceleration

- When the beam is accelerated, its longitudinal momentum is increased,
- But the transverse momentum remains the same.
- Hence the beam divergence decreases.





- Accelerating the beam leads to a reduction the volume occupied in phase space.
- This reduction is proportional to the increase of the relativistic gamma.
  => The beam size is reduced when the beam is accelerated!

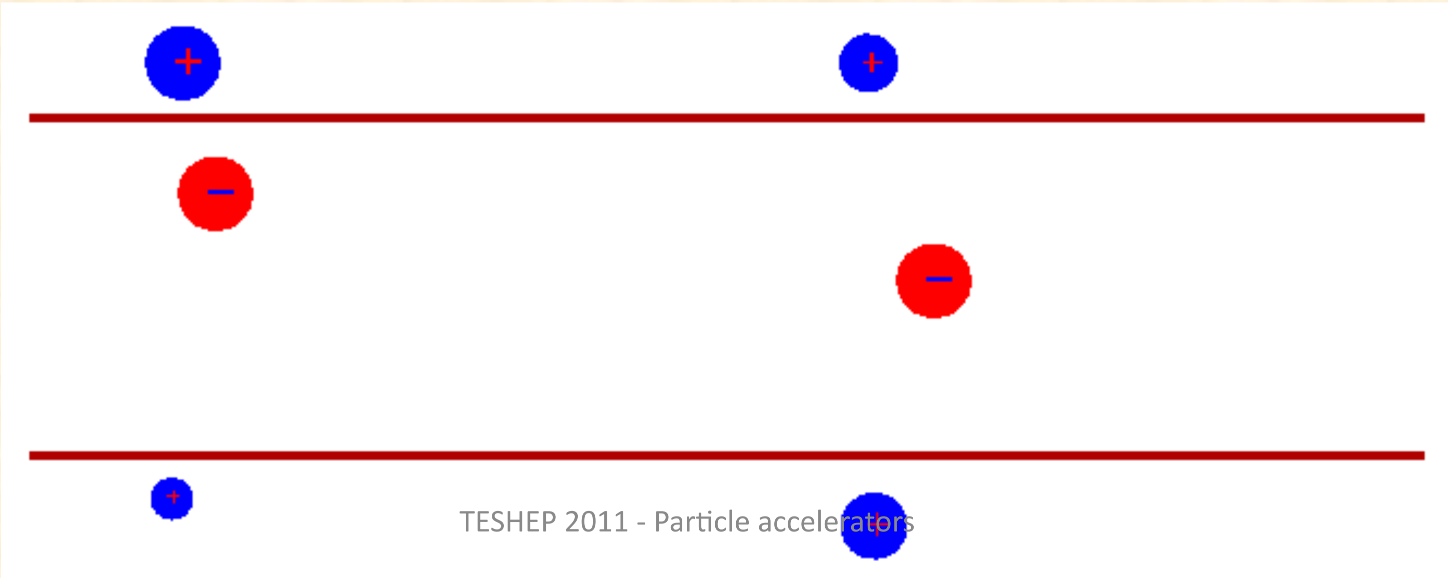
Normalised emittance

- It is convenient to define the normalised emittance of a beam: it is the volume of phase space occupied by the beam multiplied by gamma.
- The actual volume of phase space occupied by the beam is called the geometric emittance.
- The normalised emittance of a beam is constant under acceleration.

$$\in_N = \gamma \in_{Geometric}$$

Impedance issues

- Charged particles travelling near a conductor induce image charges (induced current).
- This image current dissipate power in the beam pipe (Joule effect).
- The smaller the beam pipe, the higher the induced charge and thus the highest the losses.
- The impedance of a beam pipe must be carefully controlled!



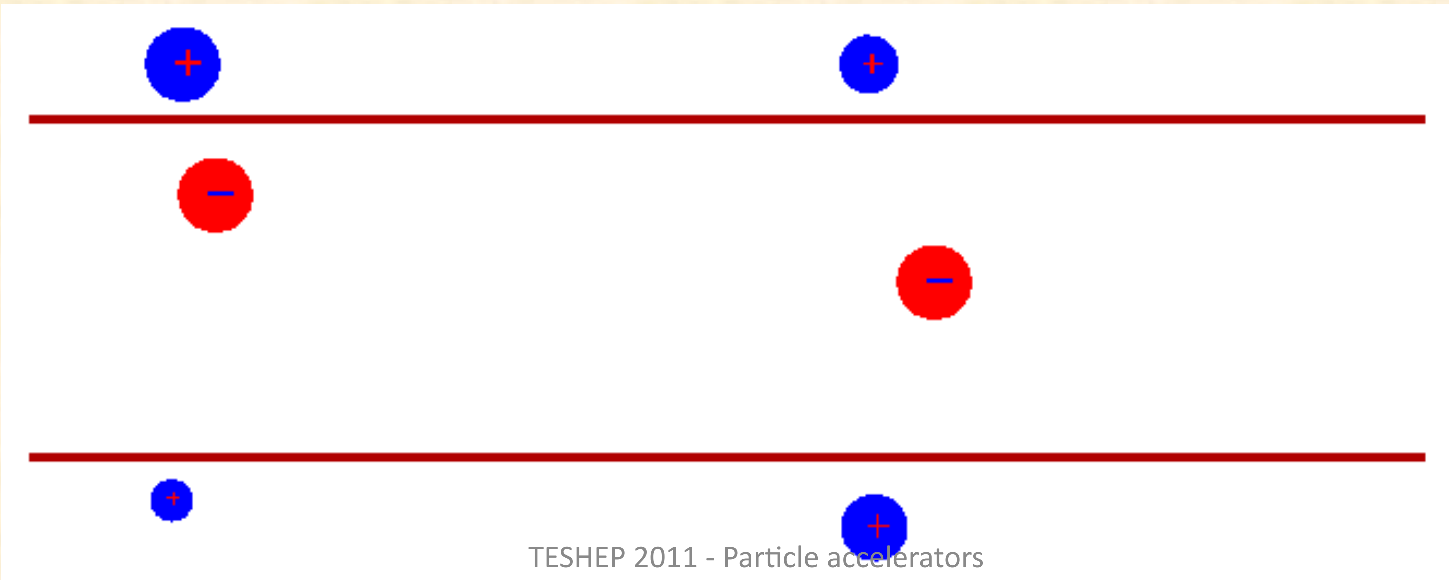
Quiz

- In the example below, which particle will create the more induced current on the beam pipe?

a) The particle on the left hand-side

b) The particle on the right hand-side

c) Both will induce the same current

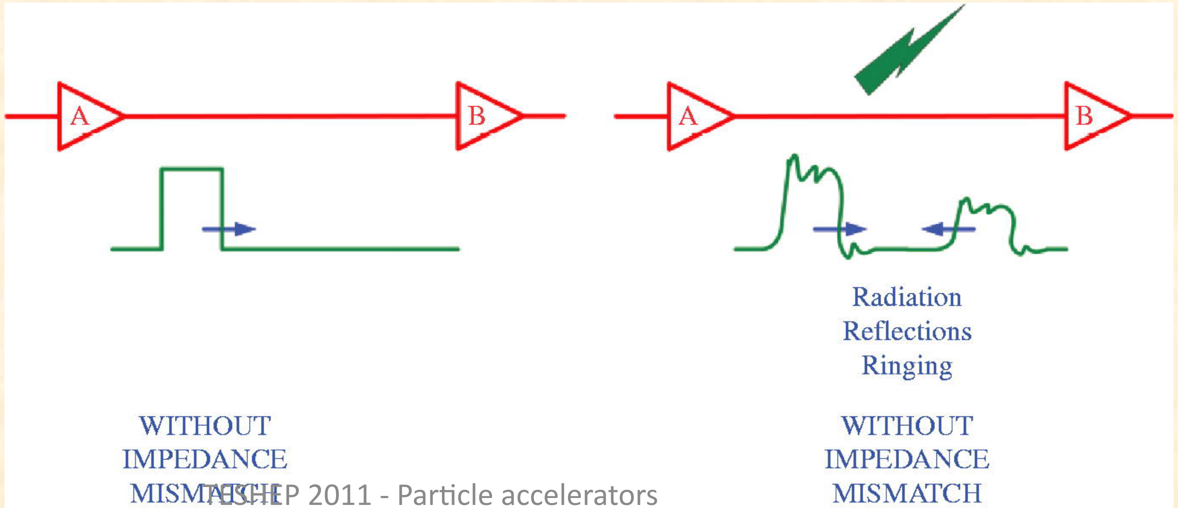


Answer (c)

- Apply Faraday's law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$,
- The total flux going through the beam pipe is the same for both particles.
- Hence the total current induced on the beam pipe by both particles will be the same.

Impedance matching

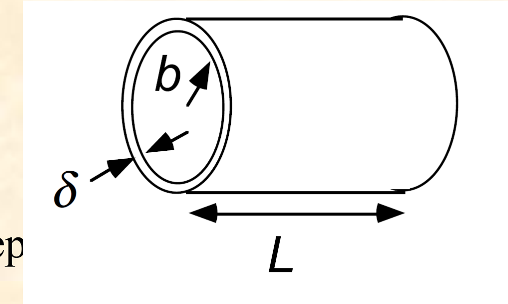
- In a RF circuit an impedance mismatch will result in a reduced transmission at the interface.
- The same is true in an accelerator: an impedance mismatch is likely to induce a reflective wave at the interface.
- This will induce a loss of power and an emittance increase.
- In a synchrotron the impedance of all beam pipe elements is carefully controlled.
- This is less important in a transfer line where the beam passes only once.



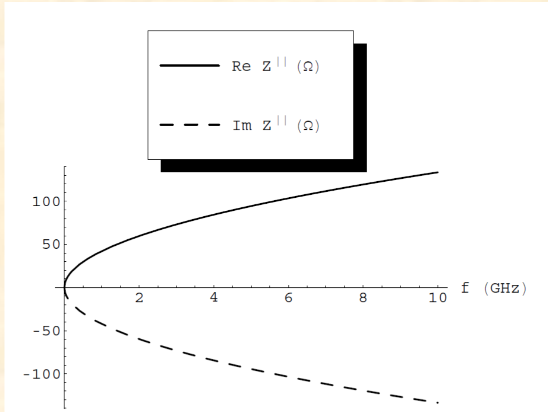
Impedance

- Let's consider a section of beam pipe.
- The impedance depends on the skin depth and the area of the pipe.
- C=ring circumference
- This impedance will induce a wake field, intense behind the beam but with a long tail
  => long distance effect for the following bunch.

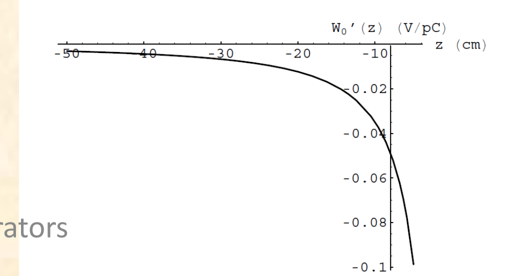
$$\delta = \sqrt{\frac{2}{\sigma\mu\omega}}$$ (skin dep



$$\frac{R_{wall}(\omega)}{L} = \frac{1}{\sigma A} = \frac{1}{\sigma 2\pi b\delta} = \frac{1}{\sigma 2\pi b\delta} = \frac{1}{2\pi b}\sqrt{\frac{\mu\omega}{2\sigma}}$$



$$W_0'(z) = -C\frac{c}{4\pi b}\sqrt{\frac{c\mu}{\pi\sigma}}\frac{1}{\sqrt{|z|^3}}$$



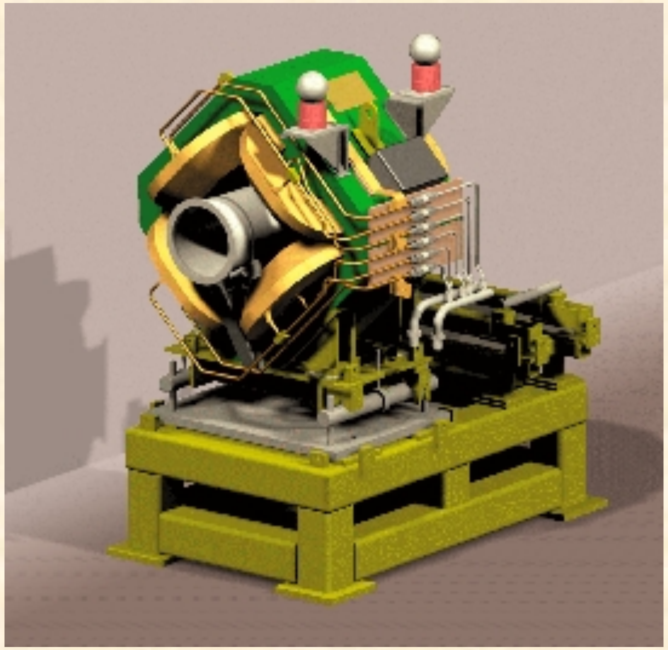
Wakefield issues

- Electrons produce an electromagnetic wave behind them.
- This can be compared the to wake of a boat and is called wakefield.
- Imagine what would happen if there was a second surfer on the picture below...
- How good is the wake for the walls of the canal?
  It is not good for the beam pipe either!

Source: Reuters

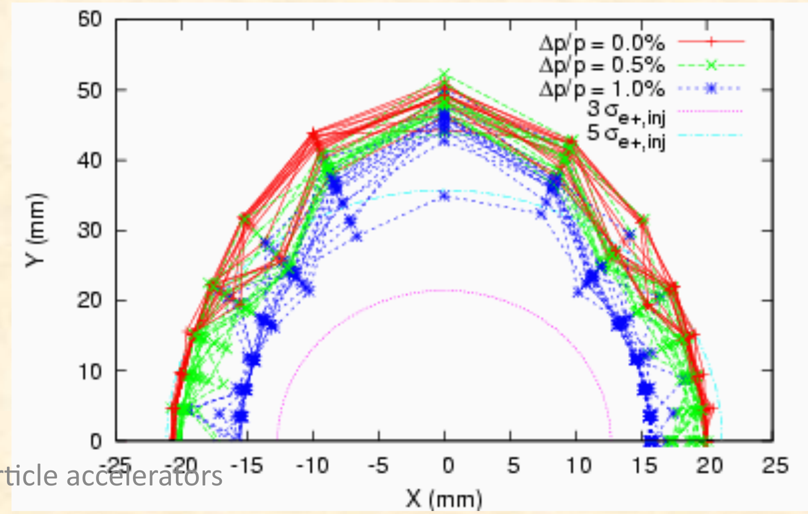
Beam size and magnet aperture

- One of the motivations for keeping the beam small is that magnets have a limited aperture.
- The larger the aperture the more difficult it is to keep a uniform field
  (and the bigger the magnet is).
- It is recommended to have a beam pipe 5 times larger than the RMS size of the beam.



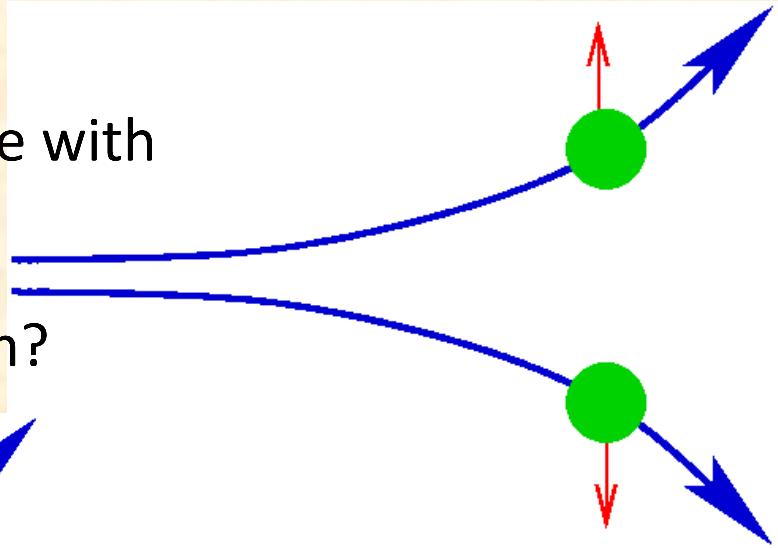
Magnet aperture

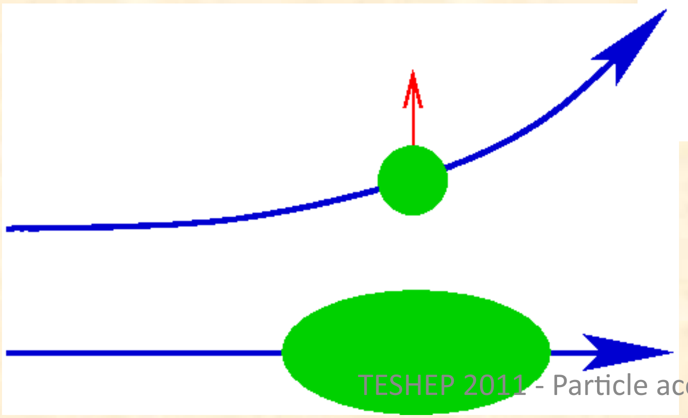
- Clipping, wakefield and impedances issues lead to poor accelerator performances.
- To avoid these effects the trajectory of the beam in the accelerator must be simulated.
- Tracking software are used to do this: they study how particles move from one location to the next.
- Instead of tracking each particle individually it is enough to track the envelope of the beam.



Space-charge effect

- Now let's consider two particles with similar charges travelling in the same direction.
- Due to their charge these particles will push each other away (Coulomb's law).
- What is the intensity of the force with which they repel each other?
- What is the effect of a full bunch?

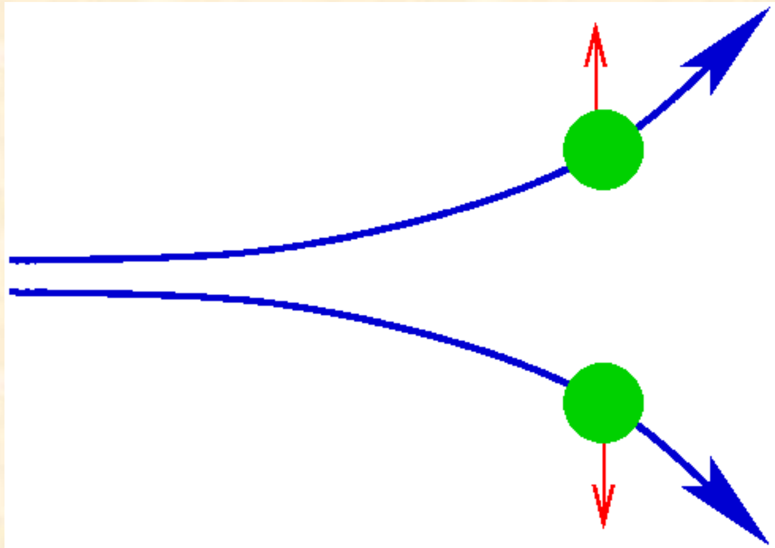


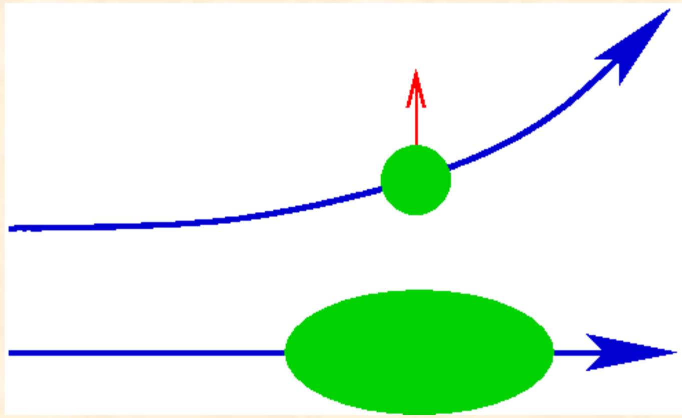


Coulomb force between two electrons

$$f = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{d^2}$$

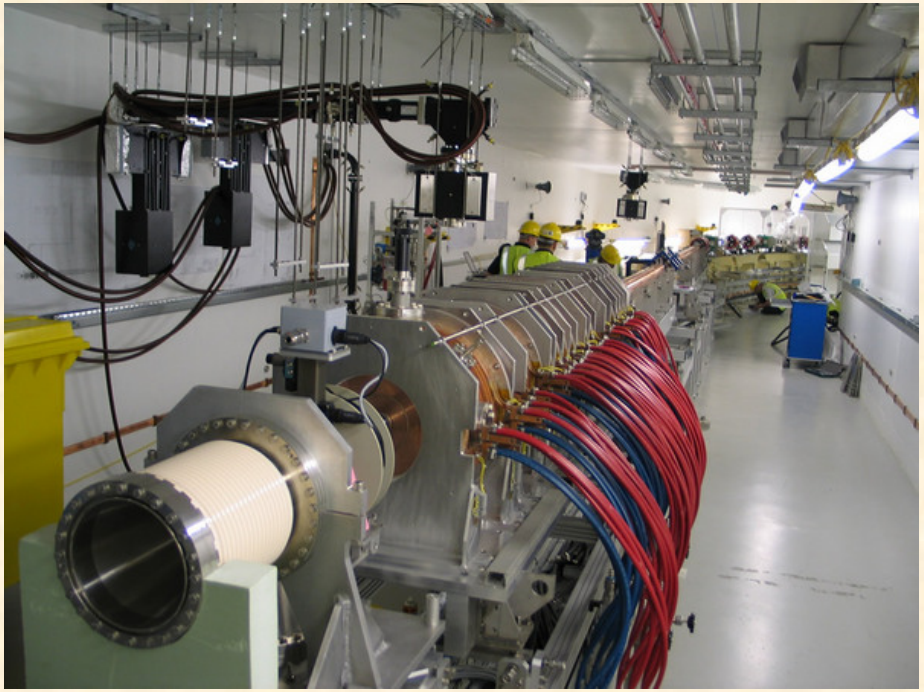
- Assume d=1micrometre.
- $f = 2\ 10^{-16}$N
- This may look small but an electron is not very heavy
- $f/m = 2.5\ 10^{14}$N/kg
- This force is very intense on the scale of the electrons.
- Typical charge in a bunch: ~100pC = 6 10^8 electrons





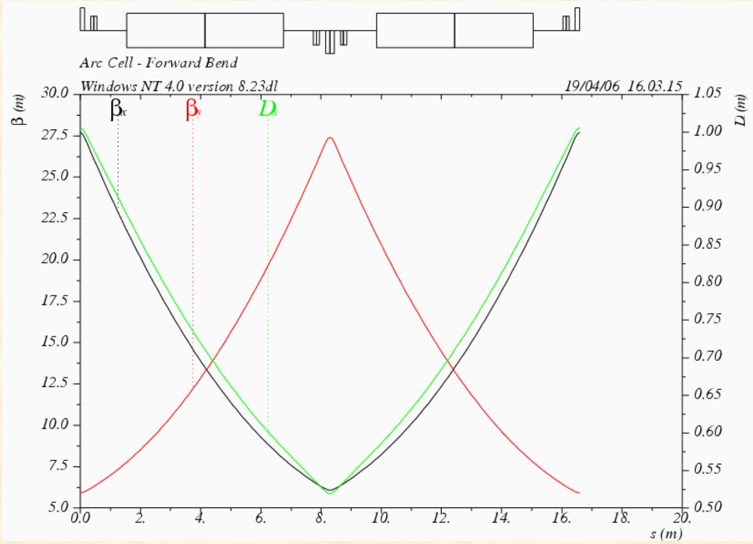
Compensating space-charge

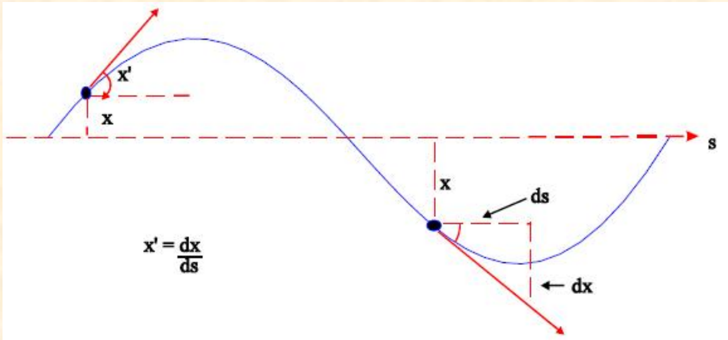
- At low energy, space-charge is a limitation of accelerators. Compensating devices have to be used to mitigate its effects.
- At high energy the "rigidity" of the beam makes space-charge less of an issue but it can not always be ignored.



Betatron oscillations

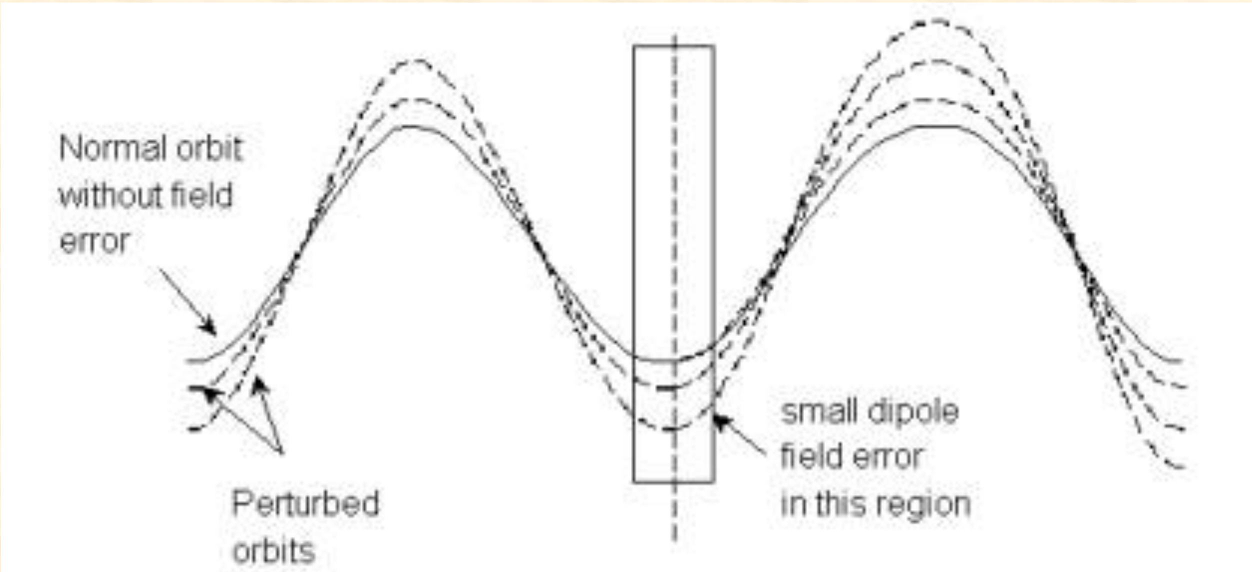
- We have seen that as it travels along the lattice the beam is focussed alternatively in both planes.
- For individual particles this leads to oscillations called "betatron oscillation".
- This occurs in both planes at the same time.
- If the particles perform an integer number of betatron oscillations in one turn, they come back at the same position turn after turn.
- Be careful, you should not confuse the number of lattice periods with the number of betatron oscillations.





Integer betatron oscillations

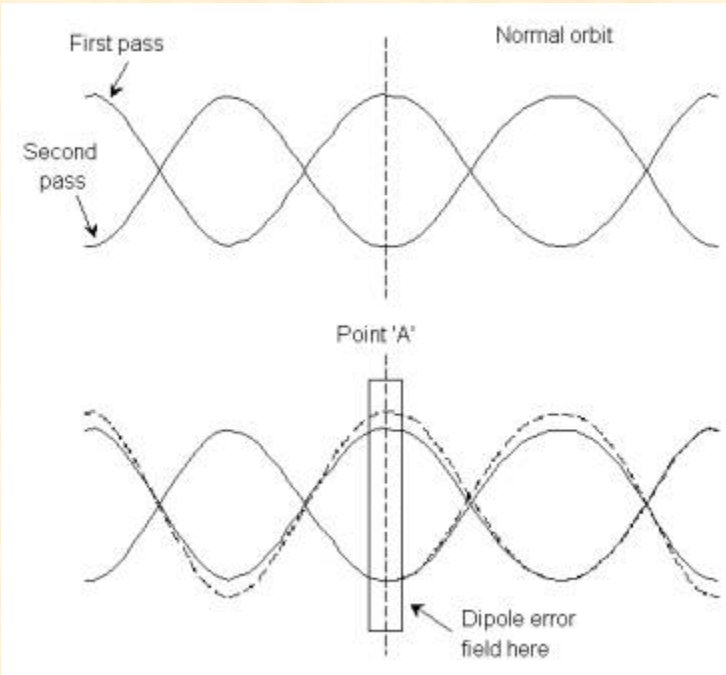
- If one of the magnets in the accelerator has a field error a particle coming turn after turn at the same position will accumulate the effect of this field error.
- After a large number of turns (millions…) what was initially a very small error can lead to large orbit changes and eventually to the loss of the beam.
  => avoid integer betatron oscillations…

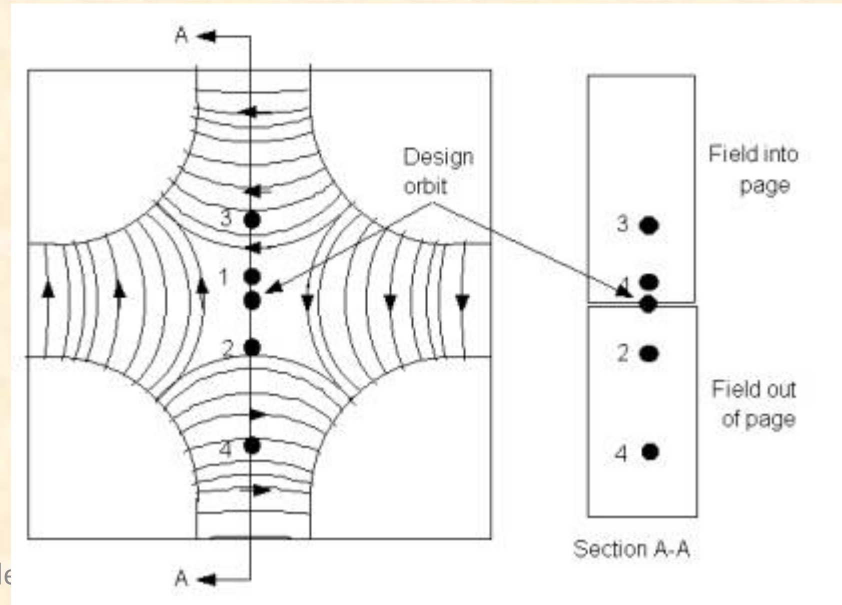


http://www.fnrf.science.cmu.ac.th/

Half-integer betatron oscillations

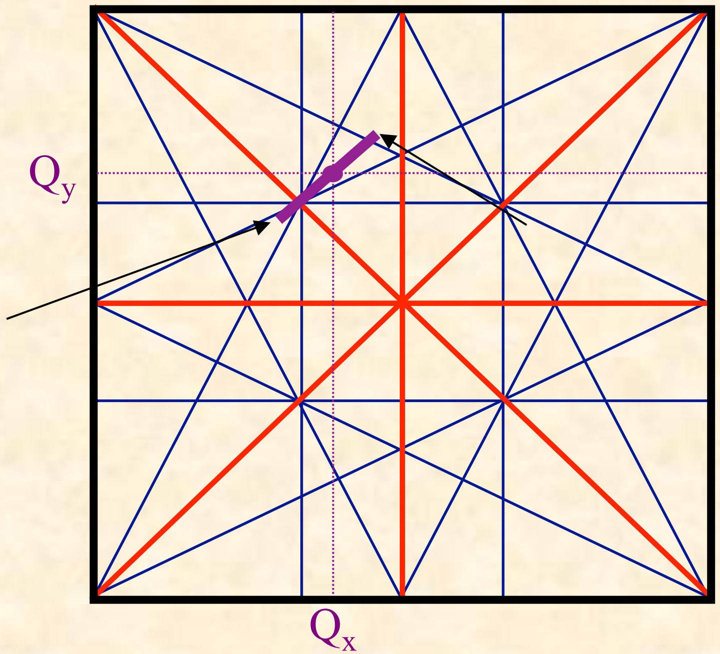
- Using an optic with an half-integer number of oscillations avoids first order problems.
- However a similar effect may accumulate every other turn…



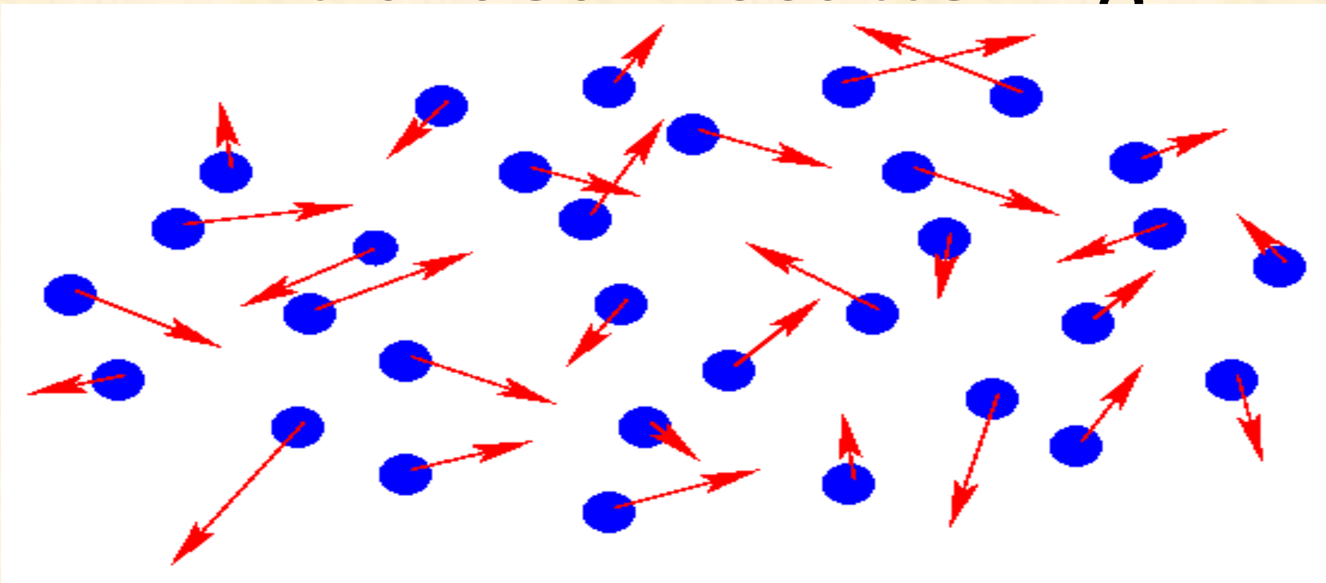


Tune

- To avoid cumulative effects in the accelerator over a large number of turns all rational numbers should be avoid for betatron oscillations in both planes
  => choose an irrational number!
- Choosing an incorrect tune can significantly increase the particle loss rate.



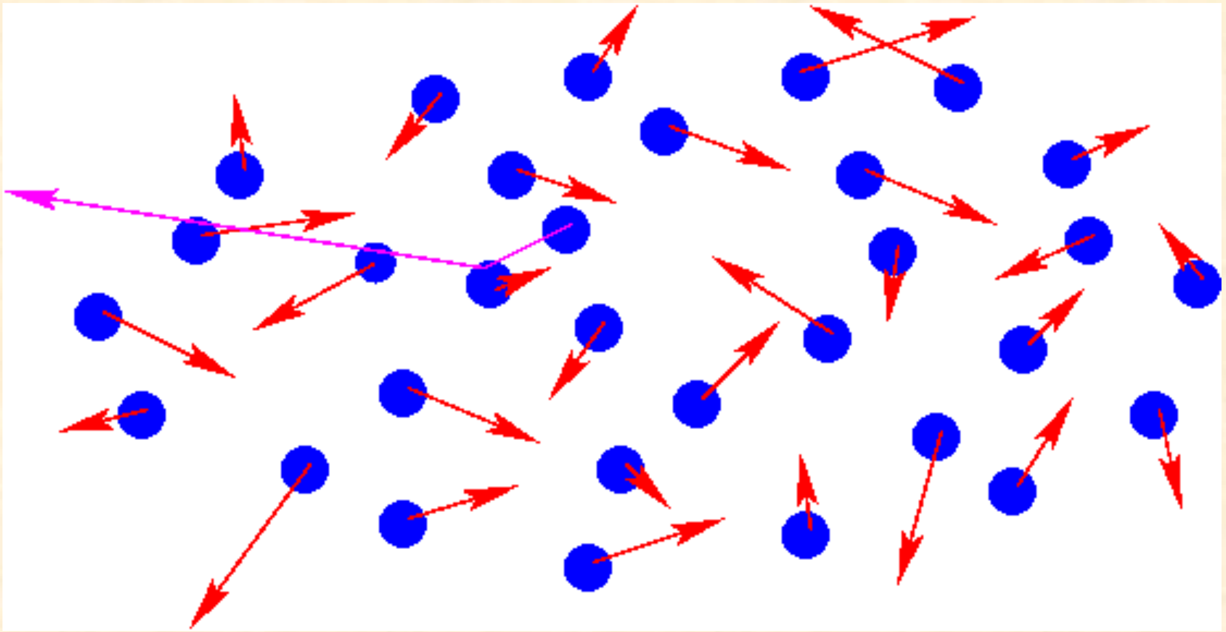
Intra-beam scattering



- We have seen that inside the beam the particles behave somewhat like a gas.
- Coulomb collisions do occur between the particles.
- These collisions lead to a momentum transfer between the particles and thus an emittance coupling and emittance growth.
- Beams with a larger emittance will experience more IBS.

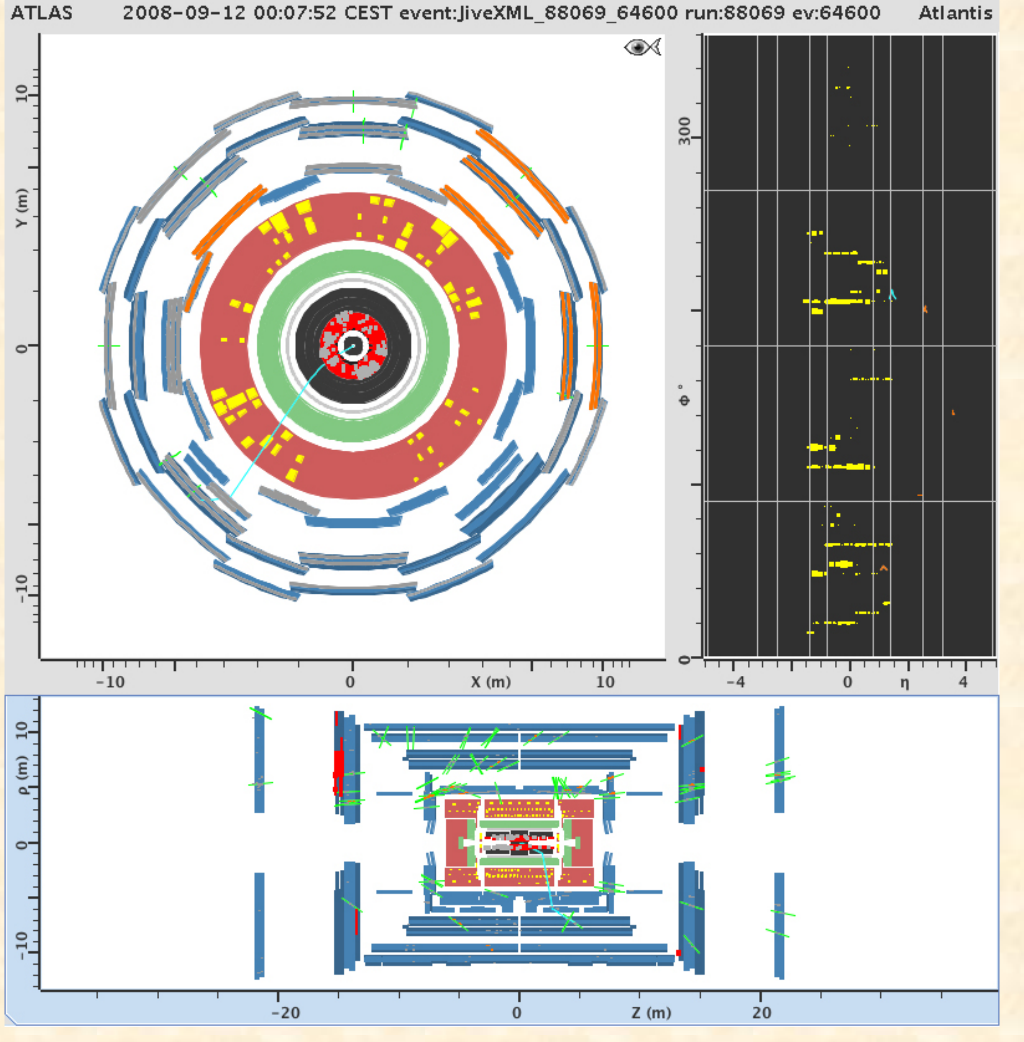
Touschek effect

- In addition to Coulomb scattering, hard scattering can also occur.
- In most cases this will lead to one particle being pushed out of the acceptable beam orbit and thus being lost soon after.
- Touschek scattering occurs in high current beams.



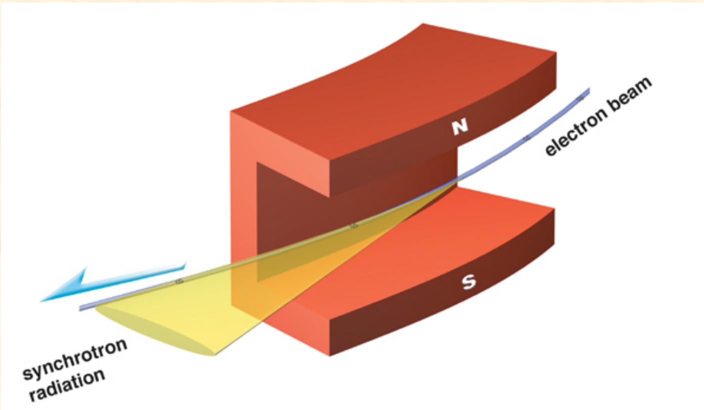
Halo

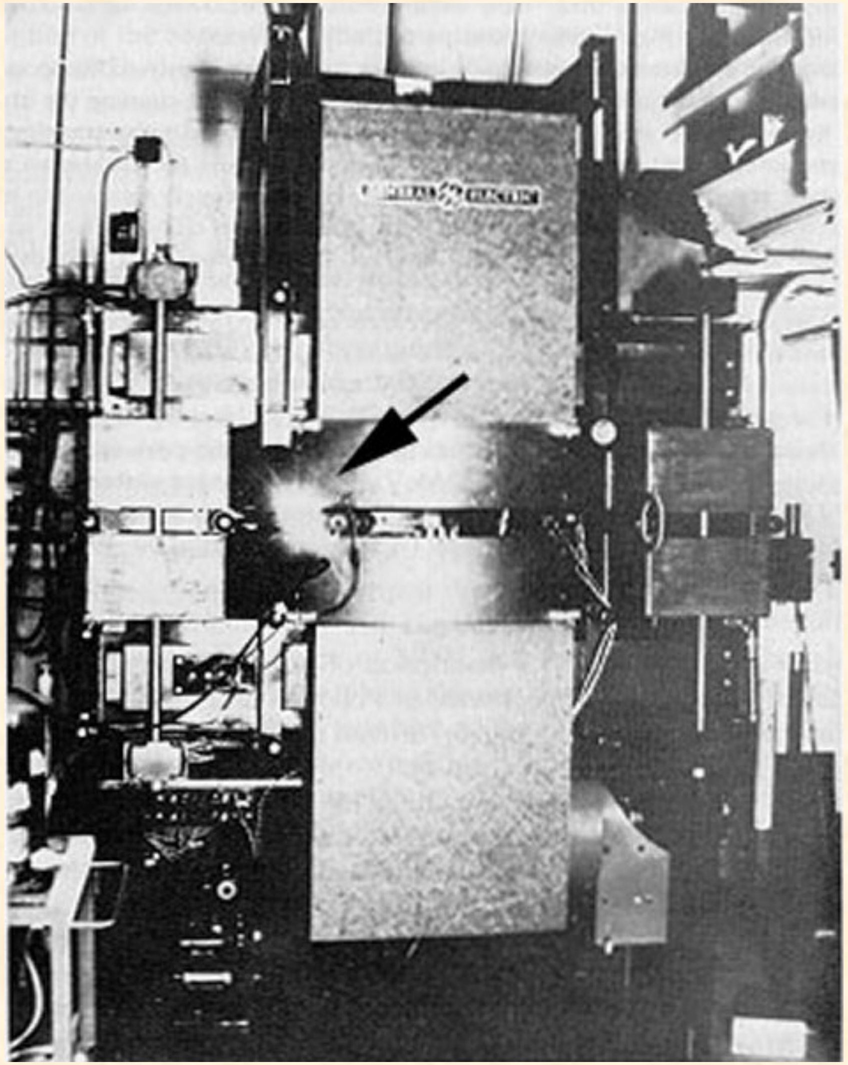
- Charged particles attract particles with the opposite charge.
- This creates a halo of particles with opposite charge around the beam.
- This halo of particles is not confined to the beam and thus it can damage equipment outside the beam pipe also.
- It is a source of beam instabilities and can lead to significant emittance growth.
- The halo must be removed by special absorbers called "collimators".



Synchrotron radiation

- Any charged relativistic particle that is accelerated or decelerated emits radiation (cf Maxwell).
- When this is a transverse acceleration as in a dipole magnet this radiation is called synchrotron radiation.



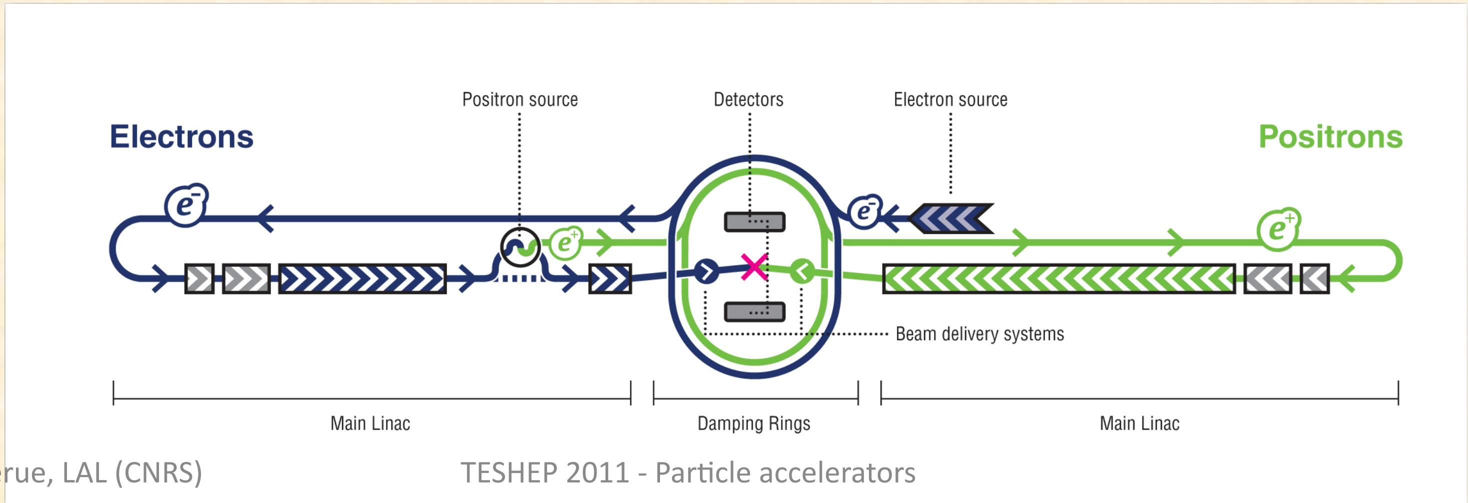


Discovery of Synchrotron radiation in 1946

Source: wikipedia

Radiation damping

- The energy lost du to synchrotron radiation emission has to be compensated by a RF cavity that tops-up the energy of the beam at each turn.
- This additional acceleration at each turns results in a decrease of the beam emittance.
- By storing a beam in a ring for several milliseconds it is possible to significantly reduce its transverse emittance.
- The reduction of emittance in a ring due to SR emission is called "radiation damping".
- As the radiation is emitted in the plane of the accelerator, radiation damping is faster in the direction orthogonal to the accelerator. => very flat beams



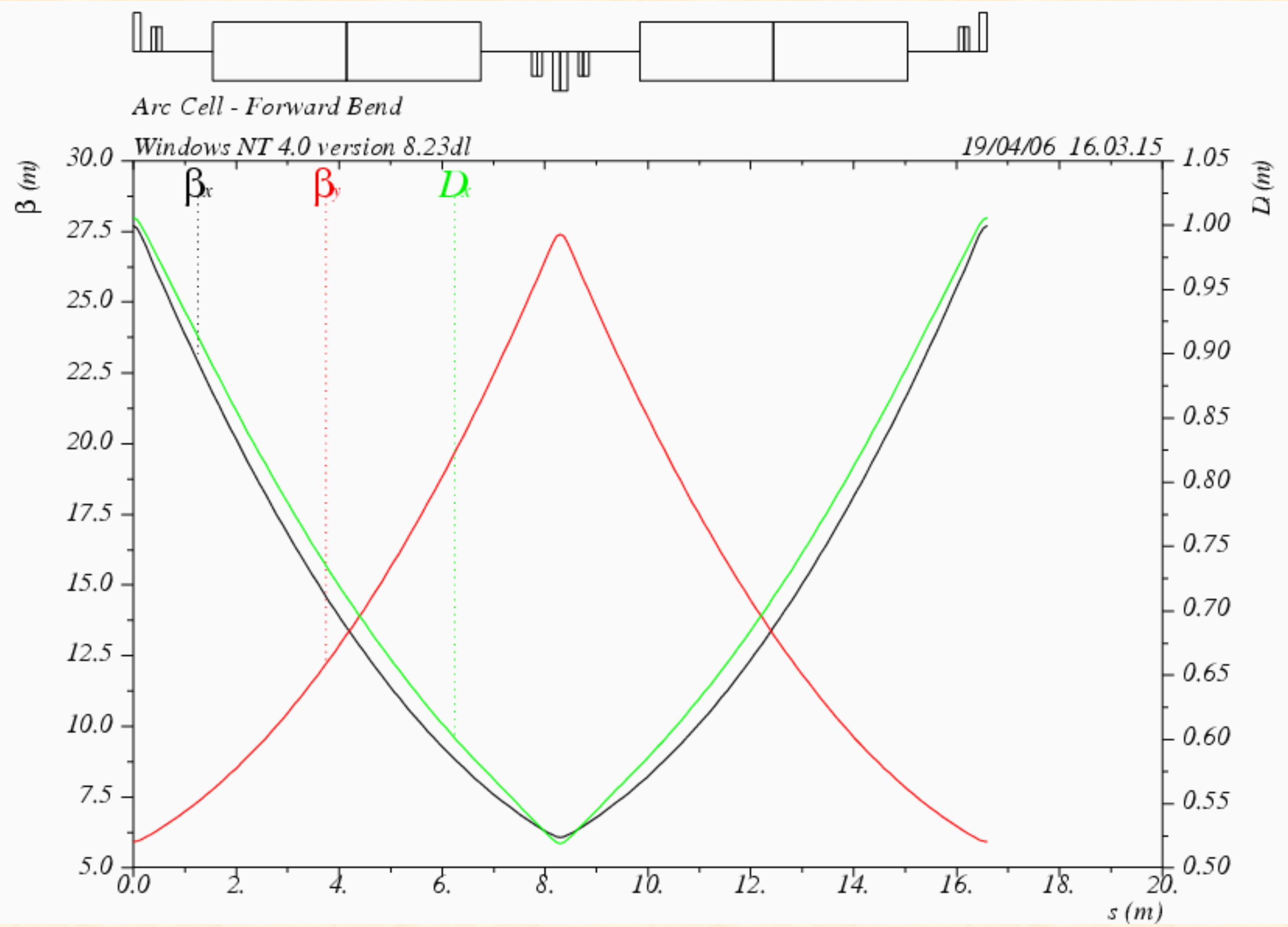
Lattice and emittance

- Radiation damping is limited by other phenomena such as emittance coupling and emittance growth.
- The minimum emittance that can be achieved depends on the latice. It is given by

$$\varepsilon_{x,\min} = \frac{F}{12\sqrt{15}} C_q \gamma^2 \frac{\theta^3}{J_x}$$

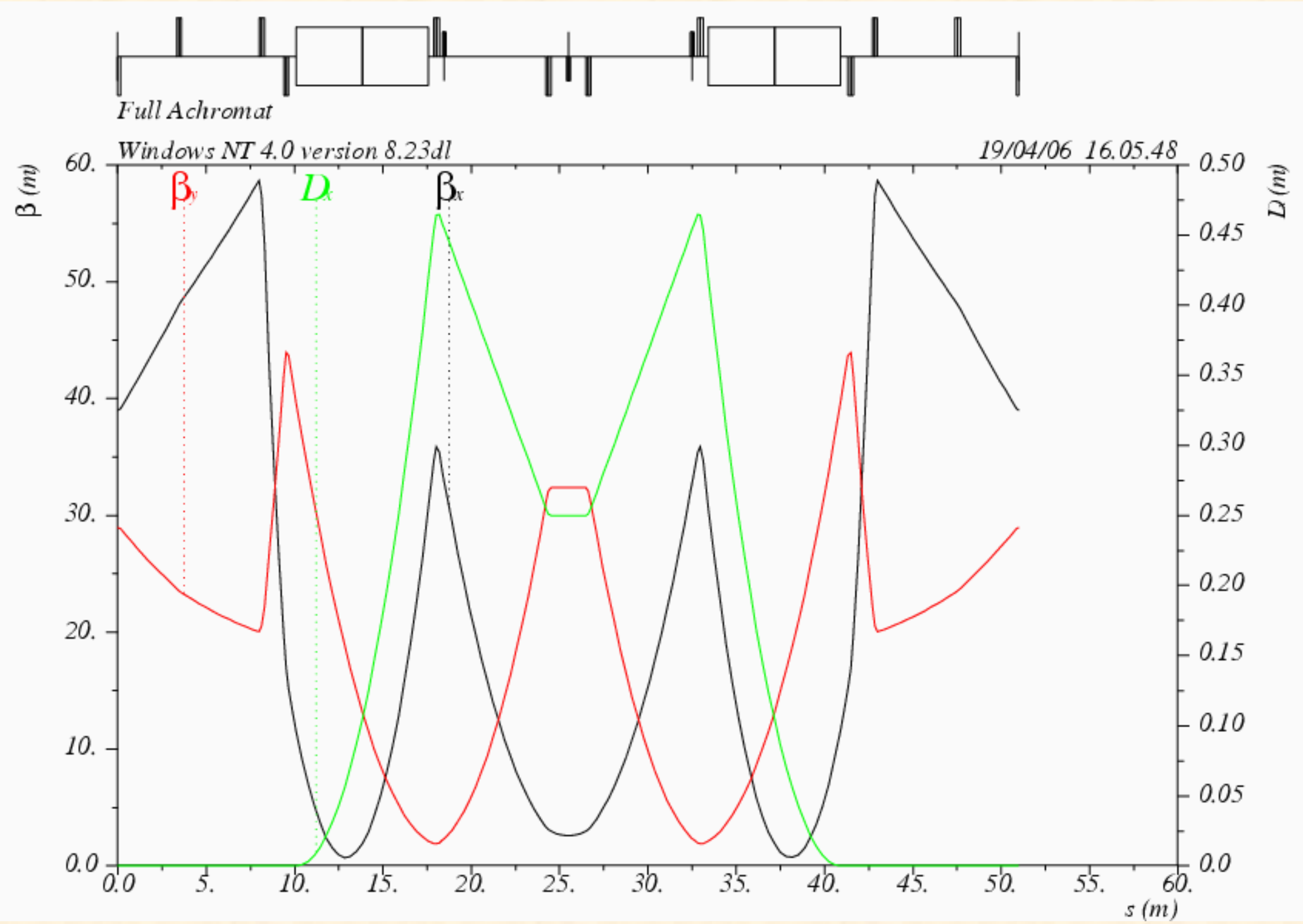
  - Where F is a factor depending on the lattice, theta the bending angle of each dipole, $C_q = 3.832\times10^{-19}$ m is a physical constant and Jx is the damping partition number.
  - To achieve a low emittance (and therefore a high luminosity) the lattice must be carefully chosen.

FODO Lattice: $F \approx 100$



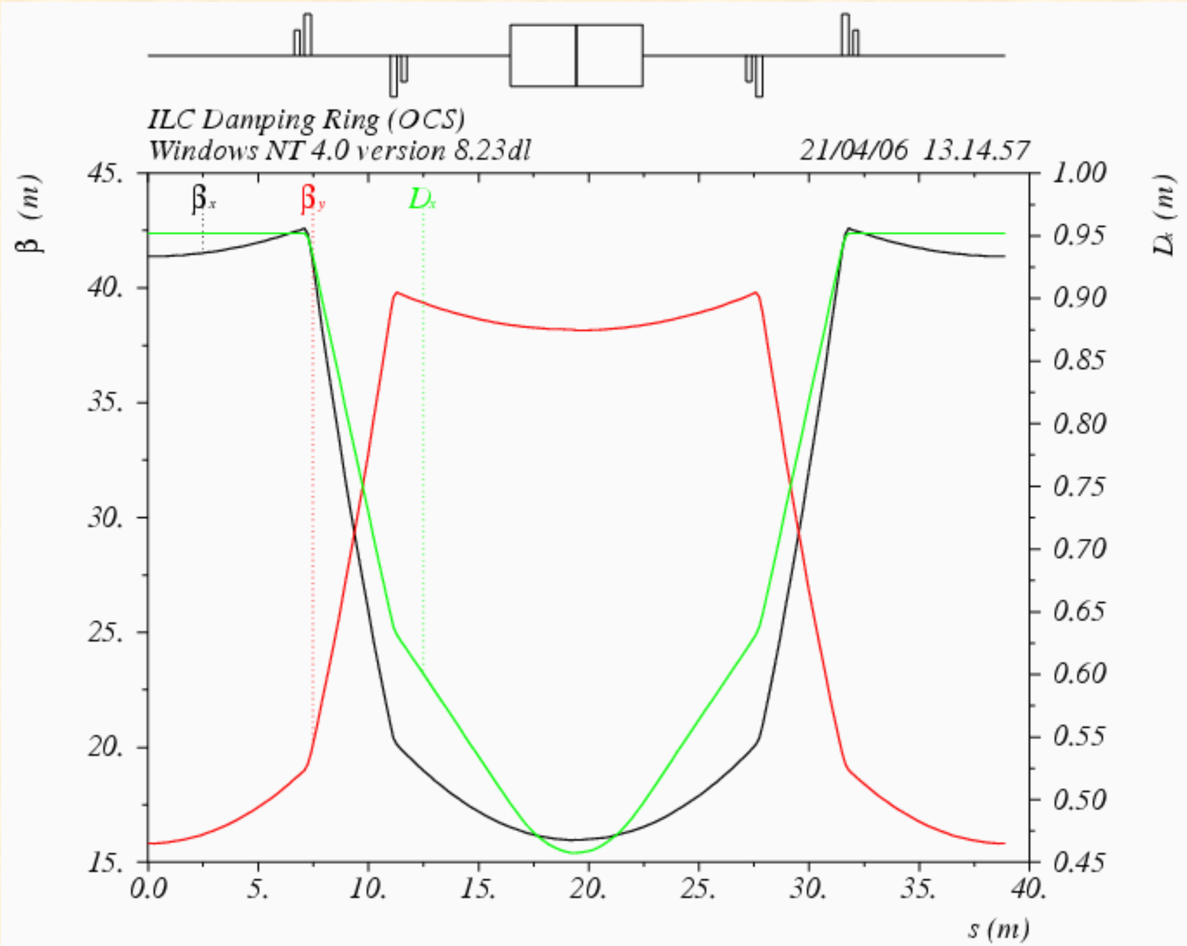
Source: Andy Wolski, ILC school

Double Bend Achromat (DBA) Lattice: *F* = 3



Source: Andy Wolski, ILC school

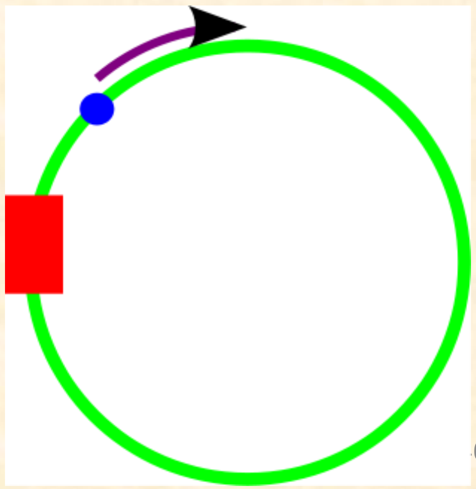
Theoretical Minimum Emittance (TME) Lattice: $F = 1$



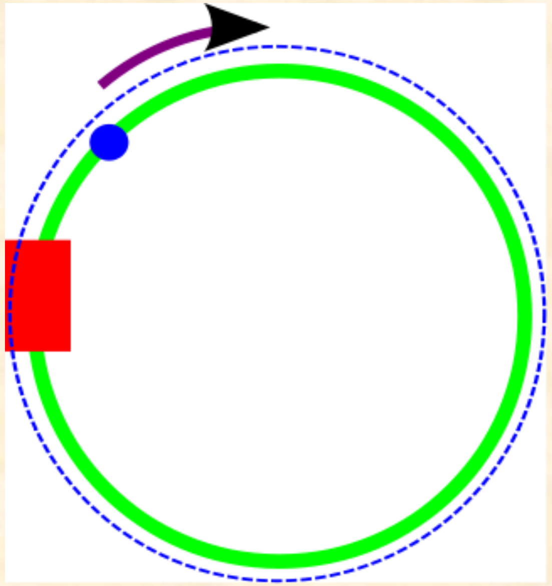
Source: Andy Wolski, ILC school

Quizz

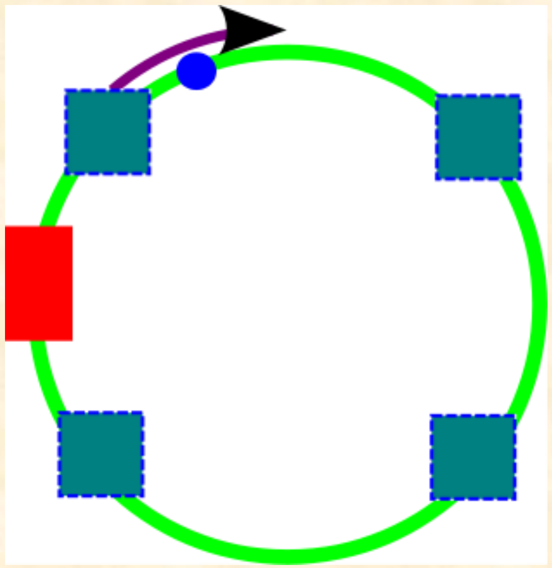
- The particles in the beam do not all have the same energy. This is the "energy spread" of the beam.
- Let's consider a ring with no focusing element (only dipoles).
- How will the trajectory of the particle with more energy compare to that of particles with less energy?
  (a) they will have a larger orbit
  (b) they will have a narrower orbit
  (c) their orbit will be similar
  (d) can not tell



Answer (a)

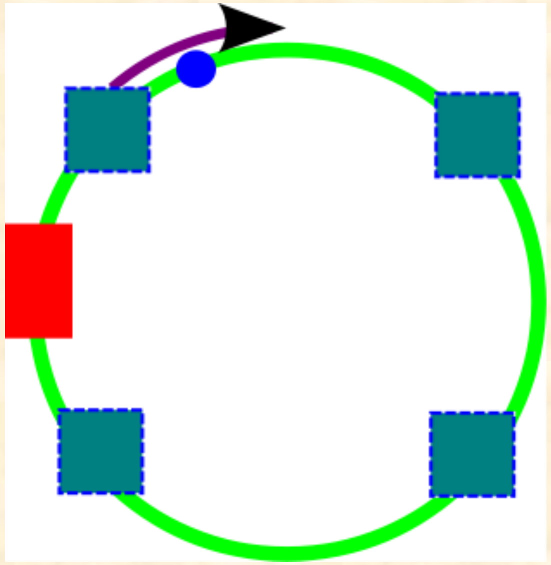


- Particles with a higher energy will be less deflected by the dipole magnets so they will have a larger orbit.
- How is this result changed if there are 4 quadrupoles in the ring?
  (a) they will have a larger orbit
  (b) they will have a narrower orbit
  (c) their orbit will be similar
  (d) Can not tell



Answer (d): Can not tell

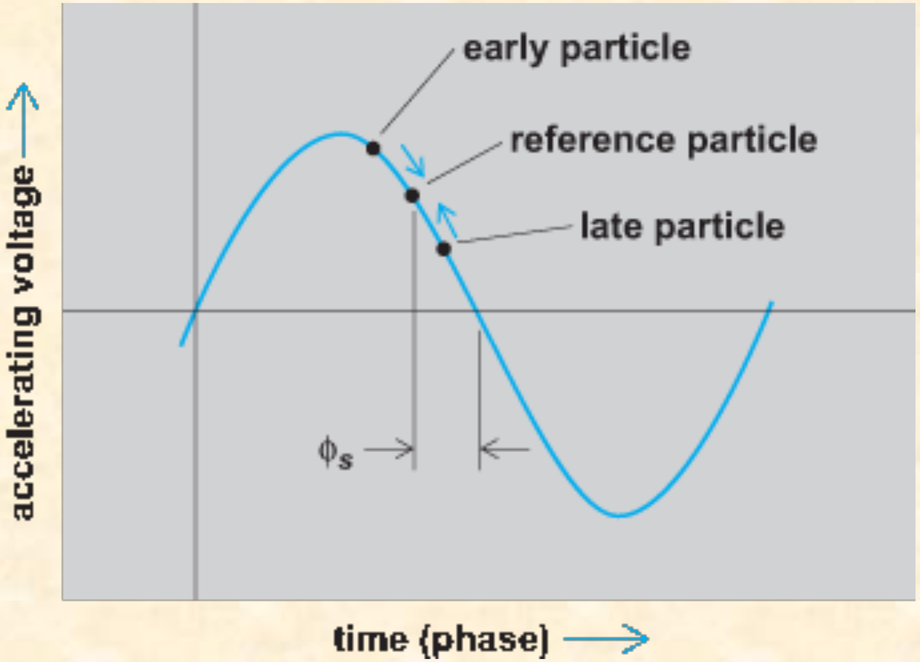
- To know which particles have a larger orbit and which particles have a narrower orbit the lattice must be taken into account.
- The momentum compaction factor gives the relation between momentum and orbit radius.
- It is dependant on the beam optics.

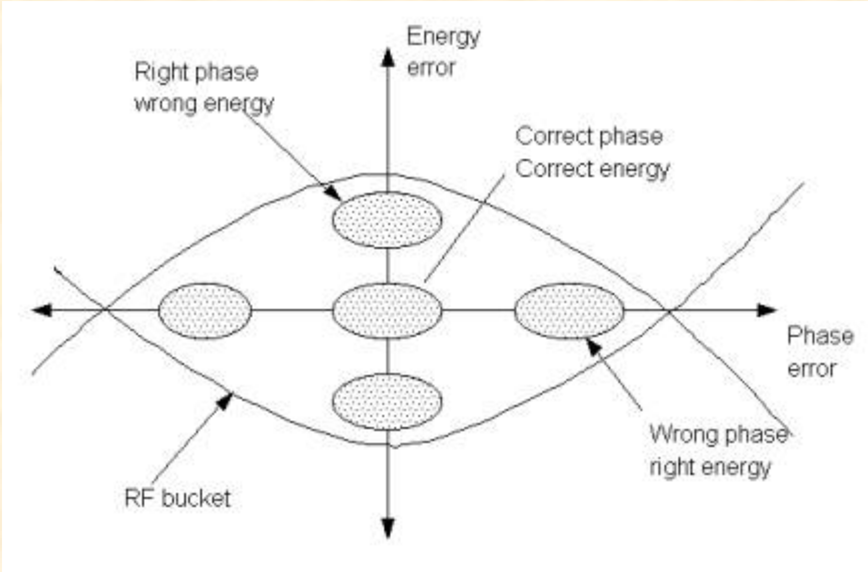


$$\frac{\Delta r}{r} = \alpha p \frac{\Delta p}{p}$$

Longitudinal dynamics

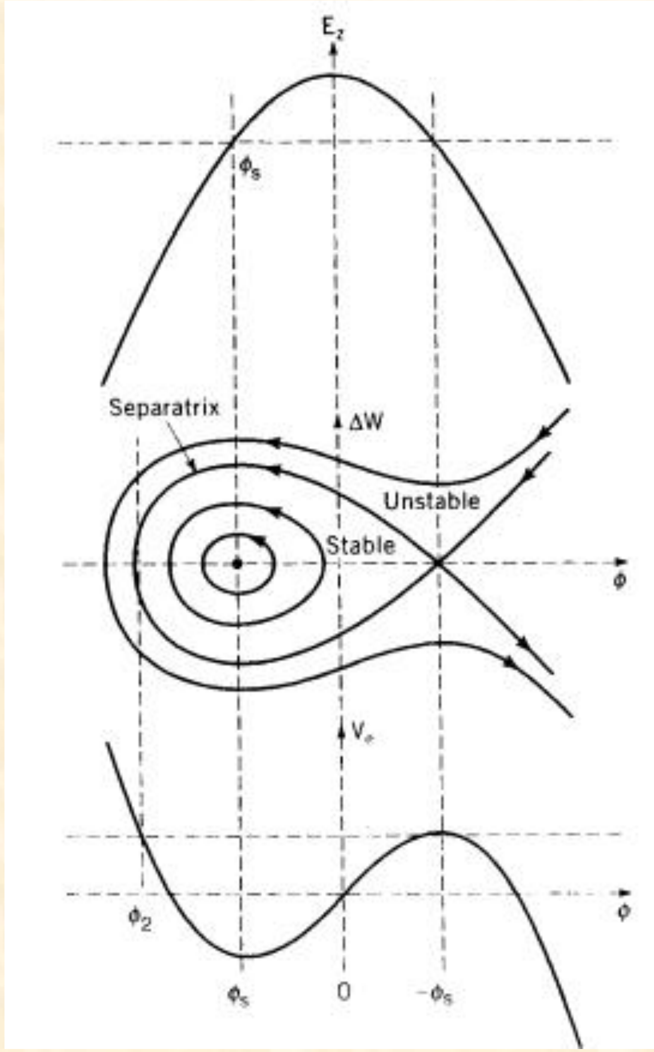
- As there is a relation between momentum and orbit, the particles that do not have the correct momentum will experience a phase slippage.
- This will in turn induce a change in their energy.





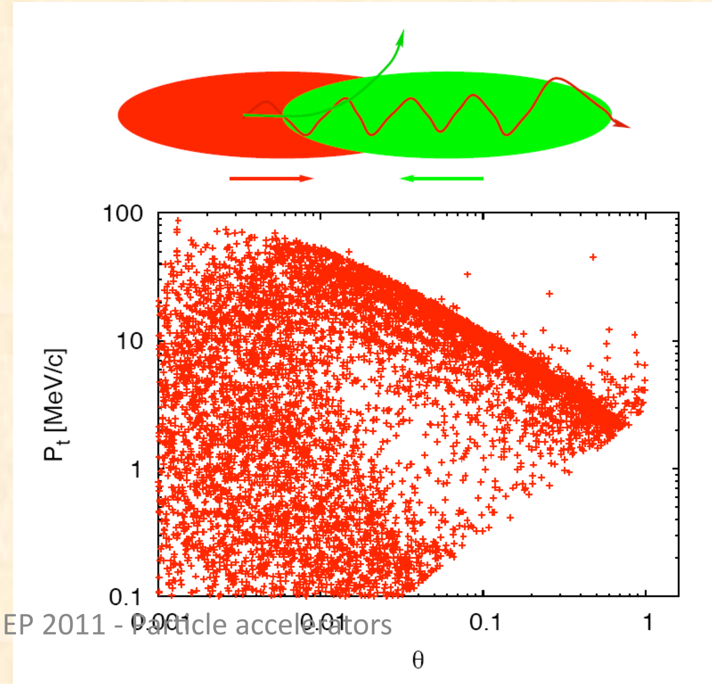
Longitudinal dynamics (2)

- Only a certain area of the phase vs energy plan is stable.
- Particles outside this stable area will drift away and eventually get lost.
- The position and size of this area will depend on the RF cavity voltage, the energy lost per turn (SR) and momentum compaction factor.

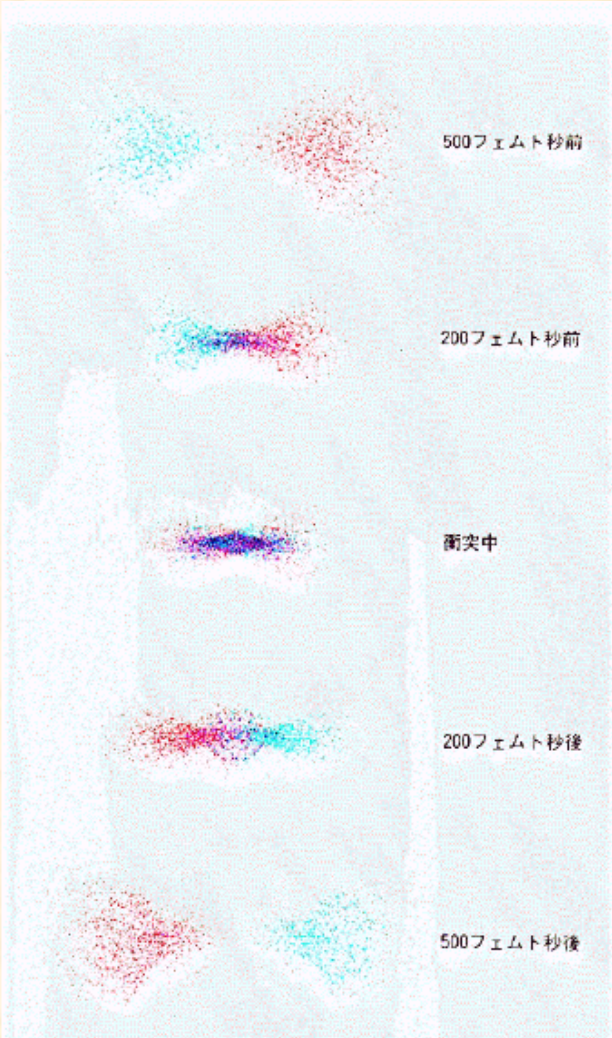


Beam beam effects (1)

- In a collider the two beams feel each other's electric field well before and well after colliding.
- Given that the particles come very close to each other, this lead to very intense forces.
- These forces lead to significant disruption of the beam.



Beam beam effect (2)



- At the interaction point the two beams self-focus onto each other.
- If the self focussing is too strong this can lead to a large emittance growth.
- If the two beam are not perfectly aligned this will also lead to large transverse deflection.
- This is a strong limitation on the size of the beams and therefore the luminosity, especially in a ring.

Summary

- The volume occupied by the beam in the 6D phase space is called its emittance.
- In an accelerator the particles interact with each other and with their environment in ways that limit the performances of the accelerator.
- Careful attention must be paid to these effects when designing or operating an accelerator to ensure optimal performances.

However

- We have not yet learned:
  - how it is possible to “see” what is happening inside the accelerator to prevent the effects
  - what particle are used for…
- We will see this tomorrow