

# Acceleration of particles in a plasma

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# Course outline

- Last lectures:
  - Electron sources
  - Ion sources
- Today:
  - Acceleration in a plasma

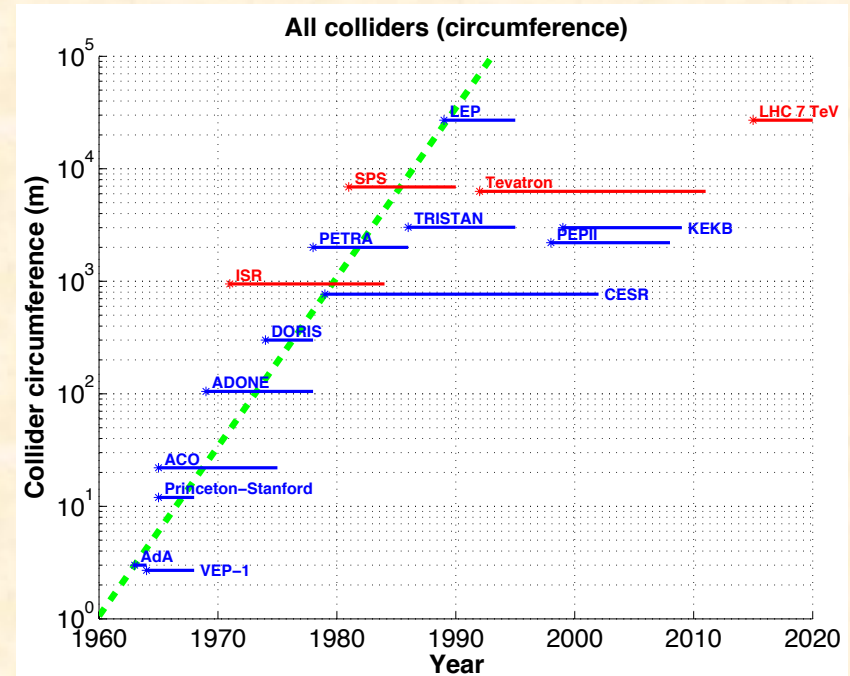
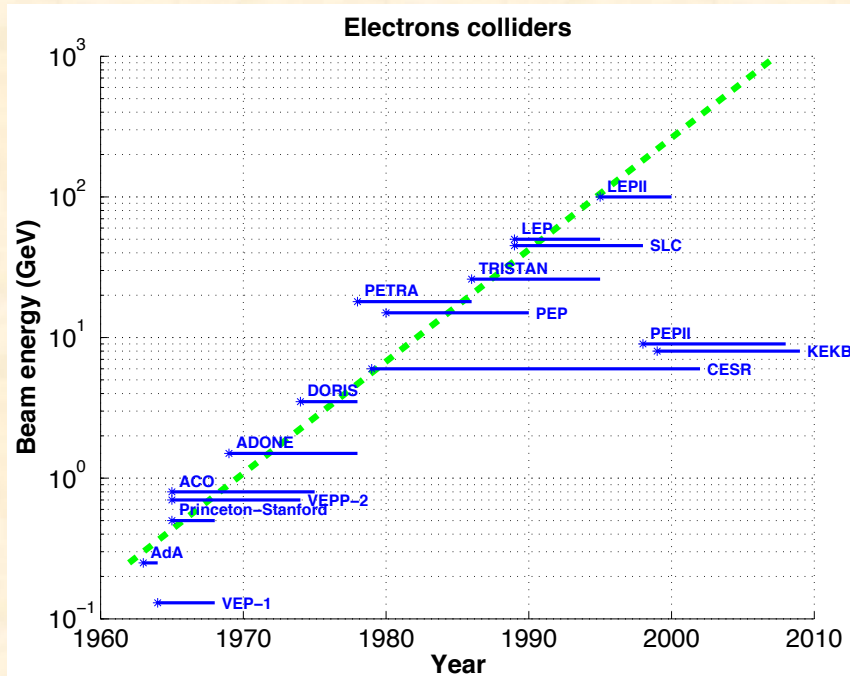
# Plasma acceleration:

## Content

- Motivation
- Acceleration of electrons in a plasma wakefield
  - Laser driven
  - Beam driven
- Acceleration of ions with a high power laser
  - The TNSA mechanism
  - Shock acceleration
- Most of the material shown here comes from the CAS School 2019 about plasma acceleration.

# Motivations:

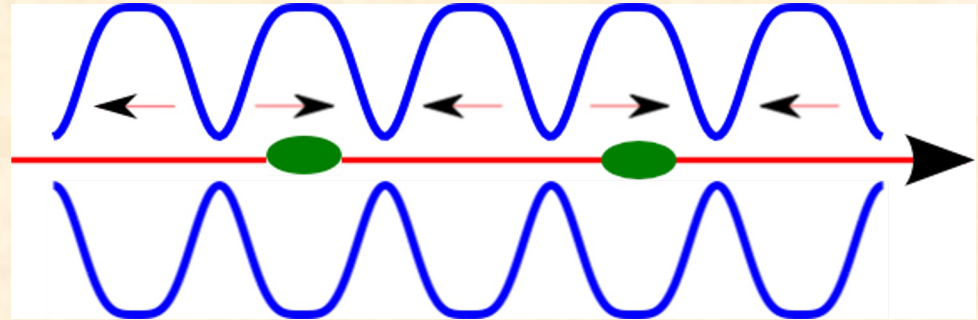
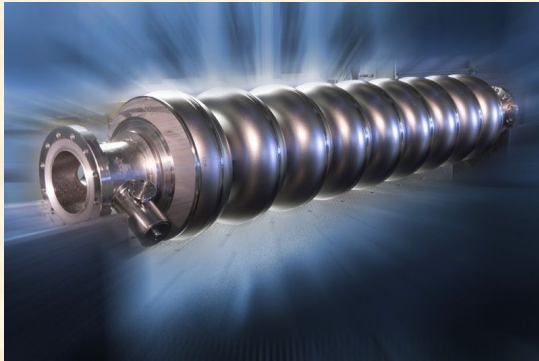
## Limits of conventional technology



- Until 1995 the centre of mass energy of lepton colliders trebled every 6 years!
- Until 1989 lepton colliders doubled their circumference every 2 years!
- Since the start of LEP II in 1995 this trend has stopped.
- Conventional technologies no longer allow significant increases of colliders' centre of mass energy at the same pace.

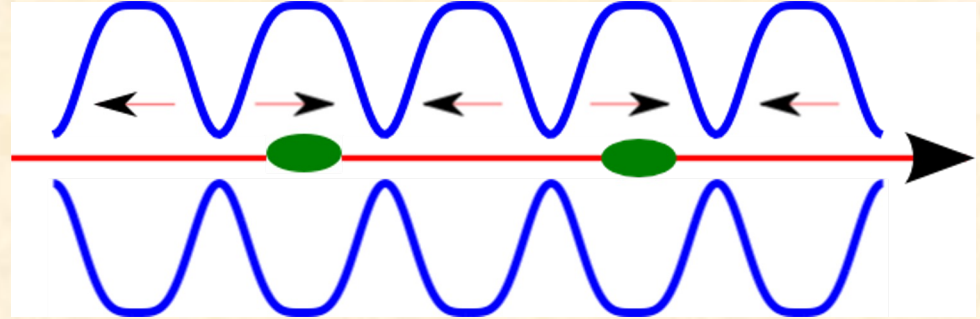
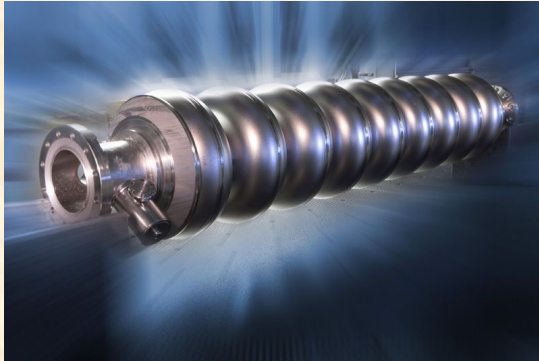


# Acceleration and RF frequency



- The highest the RF frequency, the higher the accelerating gradient will be.
- The ILC (and XFEL) operate in L-band at 1.3 GHz. Typical gradient  $\sim 20\text{ MV/m}$  (maximum  $\sim 35\text{ MV/m}$ ).  
*This corresponds to a wavelength of 23 cm.*
- The LEP injector Linac (LIL) and several conventional accelerators operated in S-band at a frequency of 3 GHz. Typical gradient (now)  $\sim 30\text{--}40\text{ MV/m}$ .  
This corresponds to a wavelength of 10 cm.
- CLIC considers operating in X-band at 12 GHz. Typical gradient  $\sim 100\text{ MV/m}$ .  
*This corresponds to a wavelength of 2,5 cm.*
- Mechanical realisation becomes more and more difficult.
- Can we do without a cavity with high frequency RF waves?

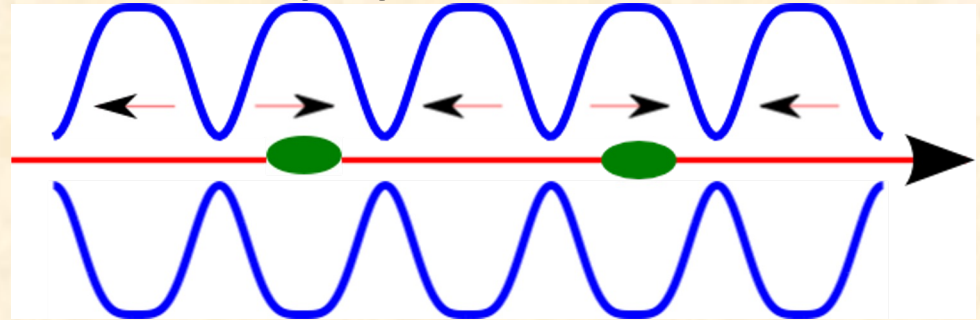
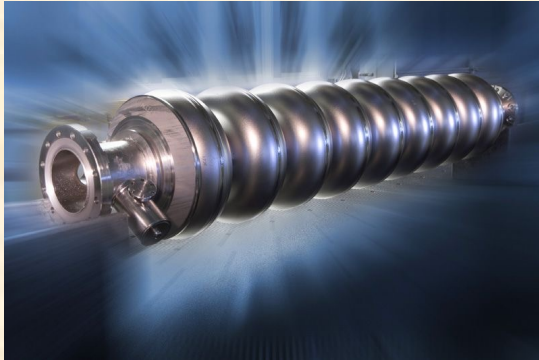
# Quizz



- What is the frequency of optical light (500nm)?
- (a) 12GHz
- (b) 100 MHz
- (c) 600THz
- (d) 3 THz

# Quizz

Answer: (C)



- What is the frequency of optical light (500nm)?
- $3\text{GHz} \Rightarrow 100\text{mm}$ ,  $50\text{mm} \Rightarrow 6\text{GHz}$ ,  $50\text{nm} \Rightarrow 6\text{PHz}$ ,  $500\text{nm} \Rightarrow 600\text{THz}$
- (a) 12GHz
- (b) 100 MHz
- (c) 600THz

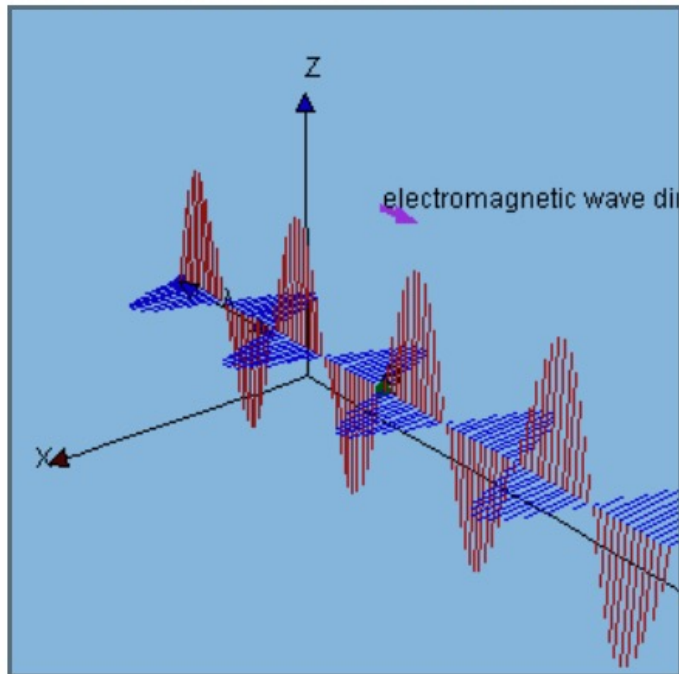
# Lawson-Woodward Theorem

(J.D. Lawson, IEEE Trans. Nucl. Sci. NS-26, 4217, 1979)

The net energy gain of a relativistic electron interacting with an electromagnetic field **in vacuum** is zero.

The theorem assumes that

- (i) the laser field is in vacuum with no walls or boundaries present,
- (ii) the electron is highly relativistic ( $v \approx c$ ) along the acceleration path,
- (iii) no static electric or magnetic fields are present,
- (iv) the region of interaction is infinite,



$$\Delta\mathcal{E} = e \int_{-\infty}^{\infty} \mathbf{v} \cdot \mathbf{E}(\mathbf{r}(t), t) dt, \quad \mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}t,$$

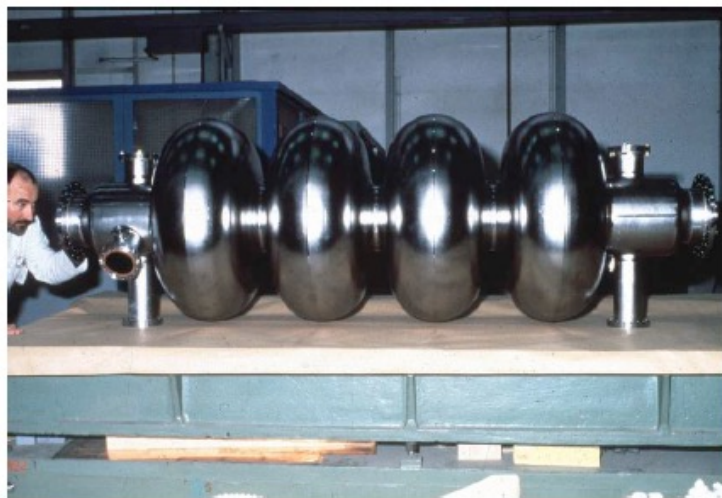
$$\mathbf{E}(\mathbf{r}, t) = \int d^3k \tilde{\mathbf{E}}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r} - i\omega t}, \quad \omega = ck.$$

$$\begin{aligned} \Delta\mathcal{E} &= e\mathbf{v} \cdot \int_{-\infty}^{\infty} dt \int d^3k \tilde{\mathbf{E}}(\mathbf{k}) e^{i\mathbf{k} \cdot (\mathbf{r}_0 + \mathbf{v}t) - i\omega t} \\ &= 2\pi e \int d^3k \mathbf{v} \cdot \tilde{\mathbf{E}}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}_0} \delta(\omega - \mathbf{k} \cdot \mathbf{v}) \equiv 0 \end{aligned}$$

$$\omega - \mathbf{k} \cdot \mathbf{v} = ck(1 - \beta \cos \alpha) > 0, \Rightarrow \delta \equiv 0$$



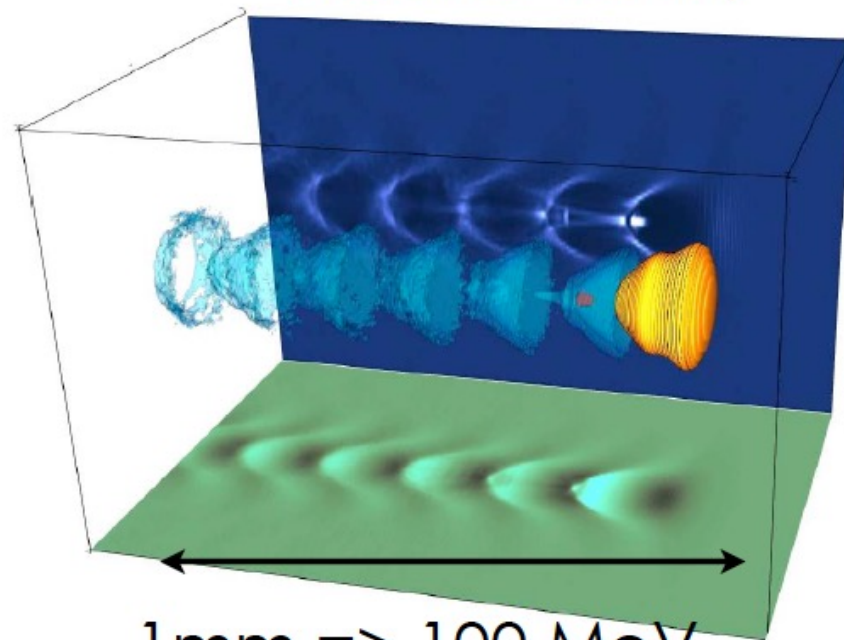
## RF Cavity



1 m  $\Rightarrow$  50 MeV Gain

Electric field  $< 100$  MV/m

## Plasma Cavity



1 mm  $\Rightarrow$  100 MeV

Electric field  $> 100$  GV/m

V. Malka *et al.*, Science **298**, 1596 (2002)

# Frequency in plasma

- Remember the characteristic oscillation frequency in a plasma:

$$\omega_e = \sqrt{\frac{e^2 n}{\epsilon_0 m_e}}$$

- For  $10^{17}$  e-/cm<sup>3</sup> this gives  $\sim 3$ THz
- Wavelength  $\sim 100\mu\text{m}$
- In an under dense plasma higher frequencies can be reached  
=> higher accelerating gradients.

# Quizz:

## Frequency in plasma

- What is the optical wavelength corresponding to 3 THz?
  - (a) 500nm
  - (b) 100um
  - (c) 500um

# Quizz:

## Answer (b)

- What is the optical wavelength corresponding to 3 THz?

~~(a) 500nm~~

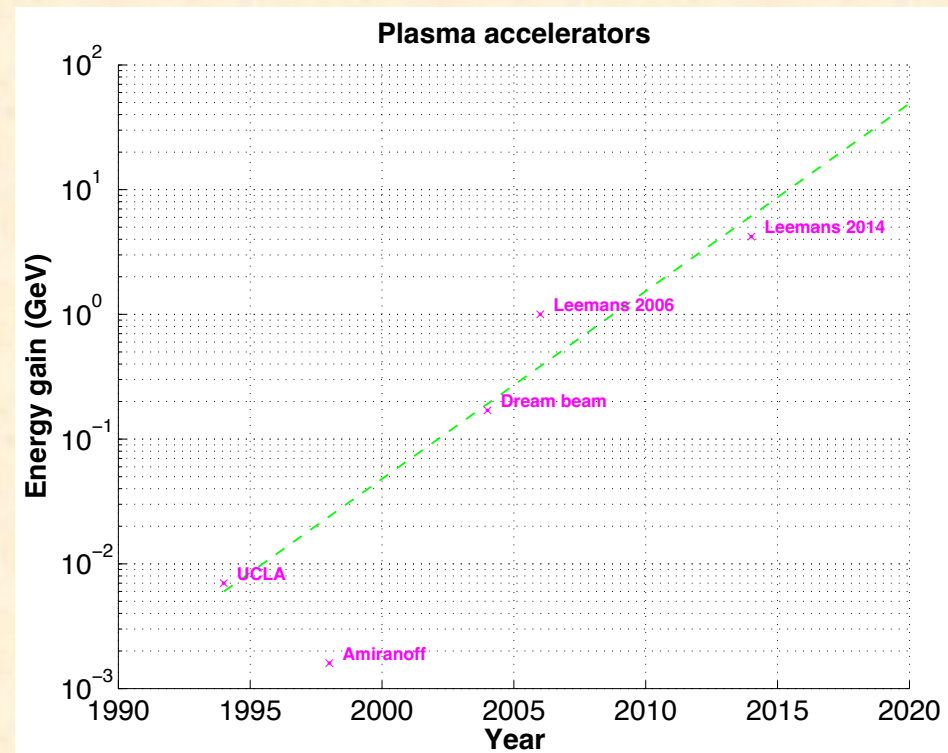
(b) 100μm (3GHz  $\Leftrightarrow$  100mm  $\Rightarrow$  3THz  $\Leftrightarrow$  100μm)

~~(c) 500μm~~



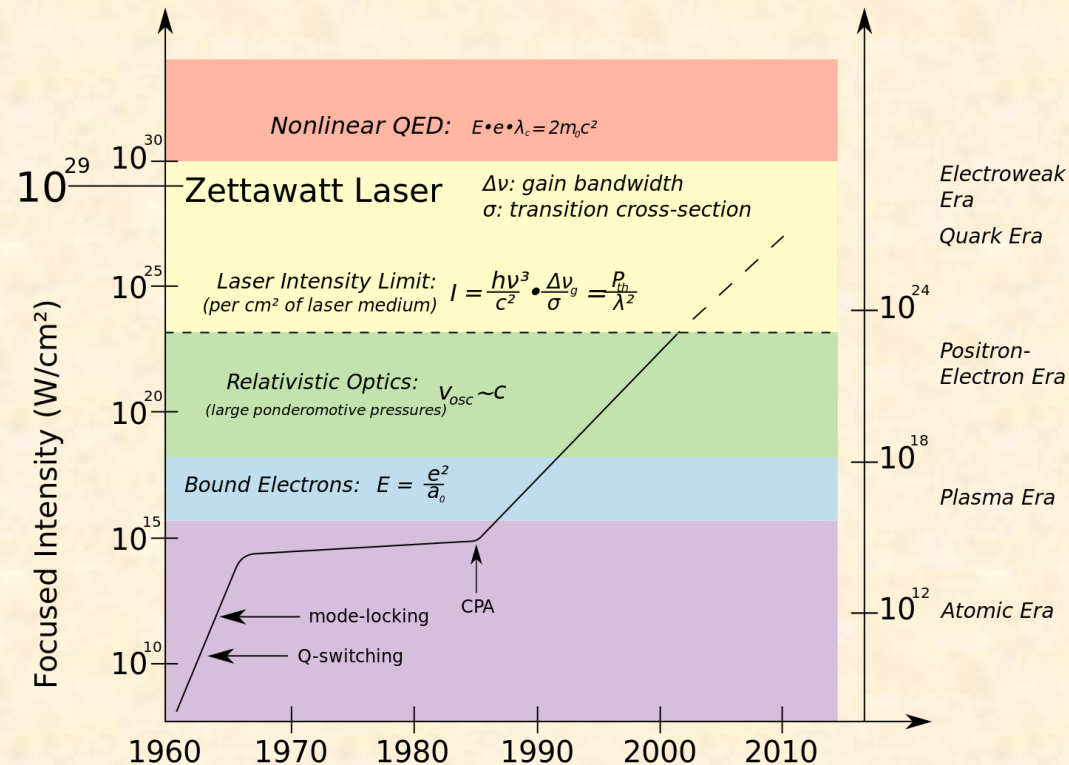
# Are laser-plasma accelerators the answer?

- Laser-plasma accelerators double the maximum energy reached every two years!
- Beware: this is the maximum energy of some particles in the beam, not the beam energy and not the energy available for HEP collisions.
- This doubling is (mostly) driven by increases in laser-power.
- Such beams are still rather unstable. They have a low charge and high dispersion with respect to what can be achieved in conventional accelerators.

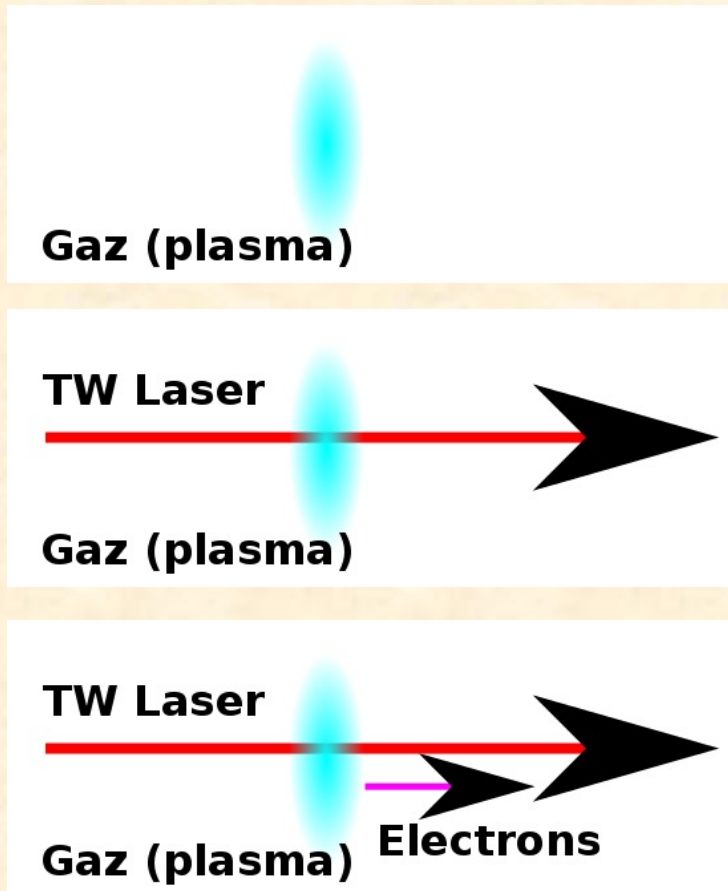


# Lasers still have a significant margin for improvement.

- Laser technology is still improving significantly.
- Fibre lasers have more and more applications with a good efficiency and they have not yet reached the “high power” range.

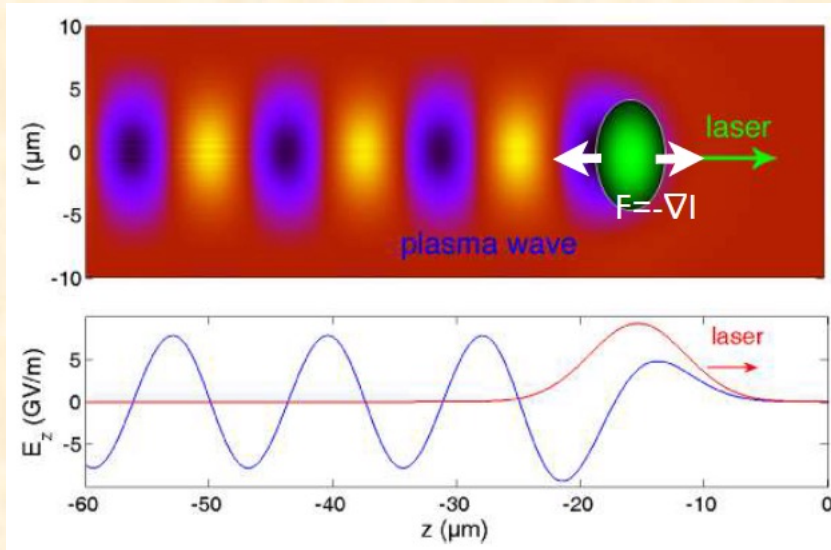


# Principe of an experiment of electron acceleration in a plasma



- A gas volume (at low density/pressure:  $\sim$ mbar) will be used to create the plasma.
- This volume is ionised by a beam (laser, particles) at high power and ultra-short (duration: ps, fs).
- Electrons coming either from the plasma or from an external source will be captured and accelerated.
- The size of the “cavities” is of the order of a few hundred micrometres.

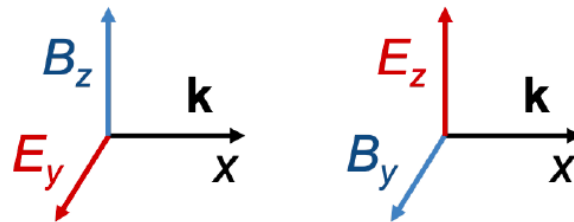
# Principle of an experiment of electron acceleration in a plasma



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- This volume is ionised by a beam (laser, particles) at high power and ultra-short (duration: ps, fs).
- Electrons coming either from the plasma or from an external source will be captured and accelerated.
- The size of the “cavities” in the wake of the laser is of the order of a few hundred micrometres.
- GV/m gradients can be reached with  $\sim 10^{18} \text{ e-}/\text{cm}^3$  ( $\sim 20 \text{ mbar}$ ).

# Electron dynamics in a plasma

## Electromagnetic plane waves



Transverse EM wave can be described by general, elliptically polarized vector potential  $\mathbf{A}(\omega, \mathbf{k})$  travelling in the positive  $x$ -direction:

$$\mathbf{A} = A_0(0, \delta \cos \phi, (1 - \delta^2)^{\frac{1}{2}} \sin \phi), \quad (16)$$

where  $\phi = \omega t - kx$  is the phase of the wave;  $A_0$  its amplitude ( $v_{os}/c = eA_0/mc$ ) and  $\delta$  the polarization parameter :

- $\delta = \pm 1, 0 \rightarrow$  linear pol.:  $\mathbf{A} = \pm \hat{\mathbf{y}} A_0 \cos \phi; \quad \mathbf{A} = \hat{\mathbf{z}} A_0 \sin \phi$
- $\delta = \pm \frac{1}{\sqrt{2}} \rightarrow$  circular pol.:  $\mathbf{A} = \frac{A_0}{\sqrt{2}} (\pm \hat{\mathbf{y}} \cos \phi + \hat{\mathbf{z}} \sin \phi)$

*Courtesy of Paul Gibbon, CAS 2019*



# Solution recipe

Bardsley et al., Phys. Rev. A 40, 3823 (1989)

Hartemann et al., Phys. Rev. E 51, 4833 (1995)

- 1 Laser fields  $\mathbf{E} = -\partial_t \mathbf{A}$ ,  $\mathbf{B} = \nabla \times \mathbf{A}$
- 2 Use dimensionless variables such that  
 $\omega = k = c = e = m = 1$   
(eg:  $\mathbf{p} \rightarrow \mathbf{p}/mc$ ,  $\mathbf{E} \rightarrow e\mathbf{E}/m\omega c$  etc.)
- 3 First integrals give conservation relations:  
 $\mathbf{p}_\perp = \mathbf{A}$ ,  $\gamma - p_x = \alpha$ , where  $\gamma^2 - p_x^2 - p_\perp^2 = 1$ ;  $\alpha = \text{const.}$
- 4 Change of variable to wave phase  $\phi = t - x$
- 5 Solve for  $\mathbf{p}(\phi)$  and  $\mathbf{r}(\phi)$

*Courtesy of Paul Gibbon, CAS 2019*

Plasma acceleration

## Solution: *laboratory* frame

Lab frame: the electron initially at rest before the EM wave arrives, so that at  $t = 0$ ,  $p_x = p_y = 0$  and  $\gamma = \alpha = 1$ .

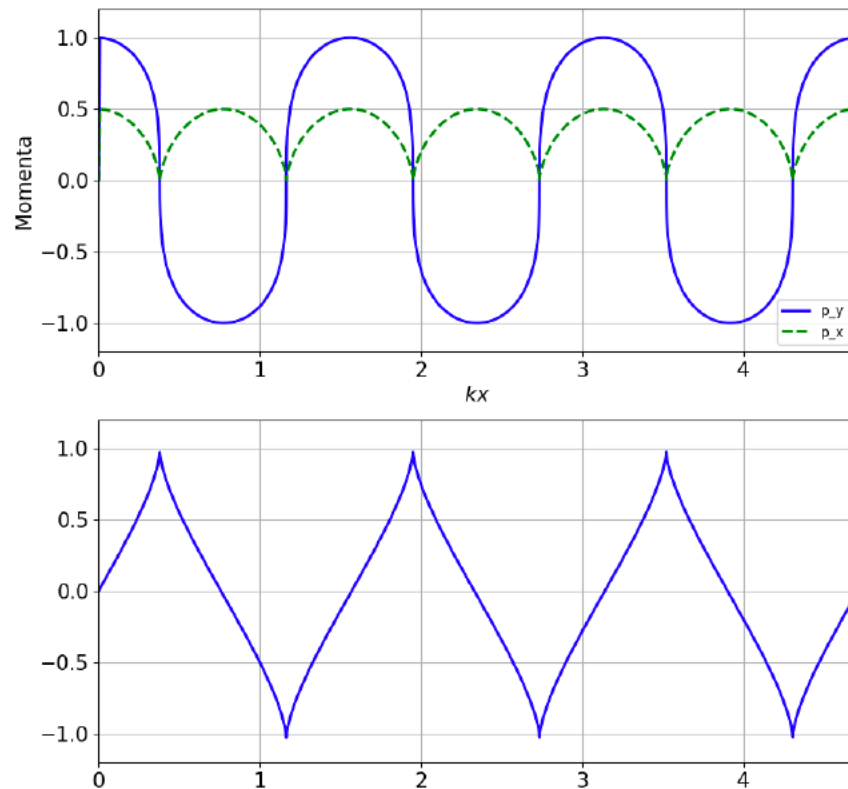
$$\begin{aligned} p_x &= \frac{a_0^2}{4} [1 + (2\delta^2 - 1) \cos 2\phi] , \\ p_y &= \delta a_0 \cos \phi, \\ p_z &= (1 - \delta^2)^{1/2} a_0 \sin \phi. \end{aligned} \tag{19}$$

Integrate again to get trajectories:

$$\begin{aligned} x &= \frac{1}{4} a_0^2 \left[ \phi + \frac{2\delta^2 - 1}{2} \sin 2\phi \right] , \\ y &= \delta a_0 \sin \phi, \\ z &= -(1 - \delta^2)^{1/2} a_0 \cos \phi. \end{aligned} \tag{20}$$

NB: solution is *self-similar* in the variables  $(x/a_0^2, y/a_0, z/a_0)$

# Linearly polarized wave ( $\delta = 1$ )



Electron *drifts* with average momentum

$$p_D \equiv \overline{p_x} = \frac{a_0^2}{4},$$

or velocity

$$\frac{v_D}{c} = \overline{v_x} = \frac{\overline{p_x}}{\gamma} = \frac{a_0^2}{4 + a_0^2}$$



## Single electron motion in EM plane wave

Electron momentum in electromagnetic wave with fields  $\mathbf{E}$  and  $\mathbf{B}$  given by Lorentz equation (SI units):

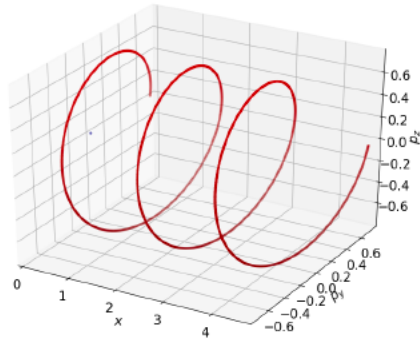
$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (17)$$

with  $\mathbf{p} = \gamma m \mathbf{v}$ , and relativistic factor  $\gamma = (1 + p^2/m^2 c^2)^{\frac{1}{2}}$ .

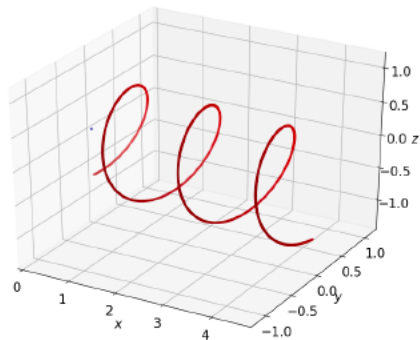
This has an associated energy equation, after taking dot product of  $\mathbf{v}$  with Eq. (17):

$$\frac{d}{dt} (\gamma mc^2) = -e(\mathbf{v} \cdot \mathbf{E}), \quad (18)$$

## Circularly polarized wave ( $\delta = \pm 1/\sqrt{2}$ )



Oscillating  $p_x$  component at  $2\phi$  vanishes, but drift  $p_D$  remains.

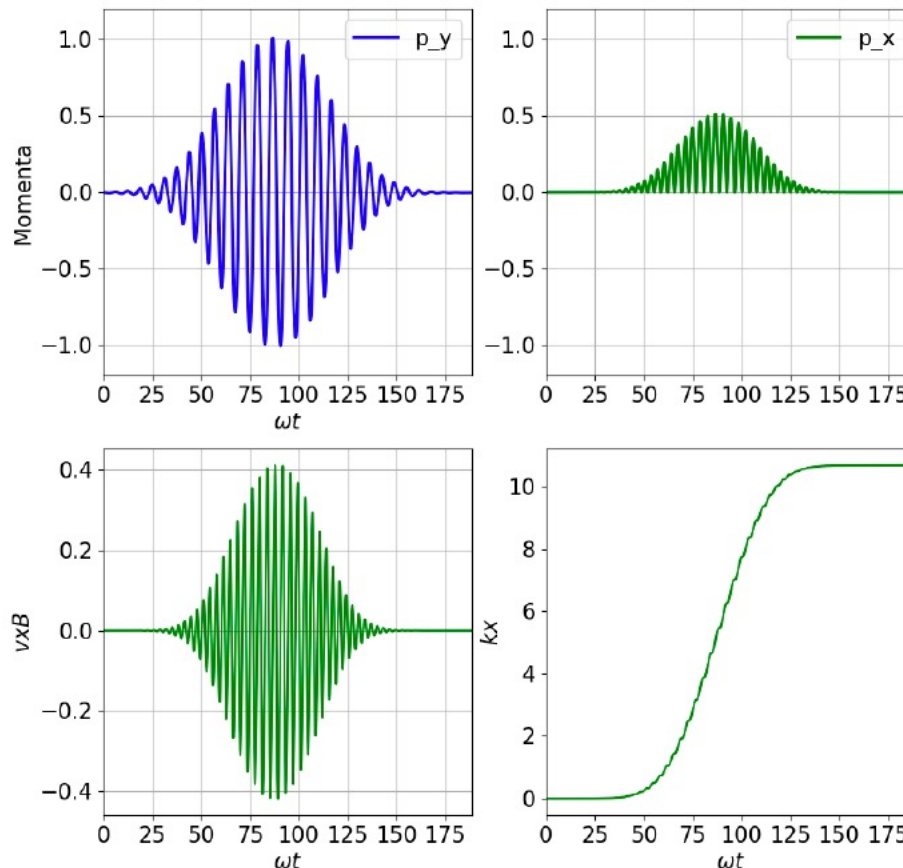


Orbit is *Helix* with:

- radius  $kr_{\perp} = a_0/\sqrt{2}$
- momentum  $p_{\perp}/mc = a_0/\sqrt{2}$
- pitch angle  $\theta_p = p_{\perp}/p_D = \sqrt{8}a_0^{-1}$

# Finite pulse duration - LP

Longitudinal Polarization



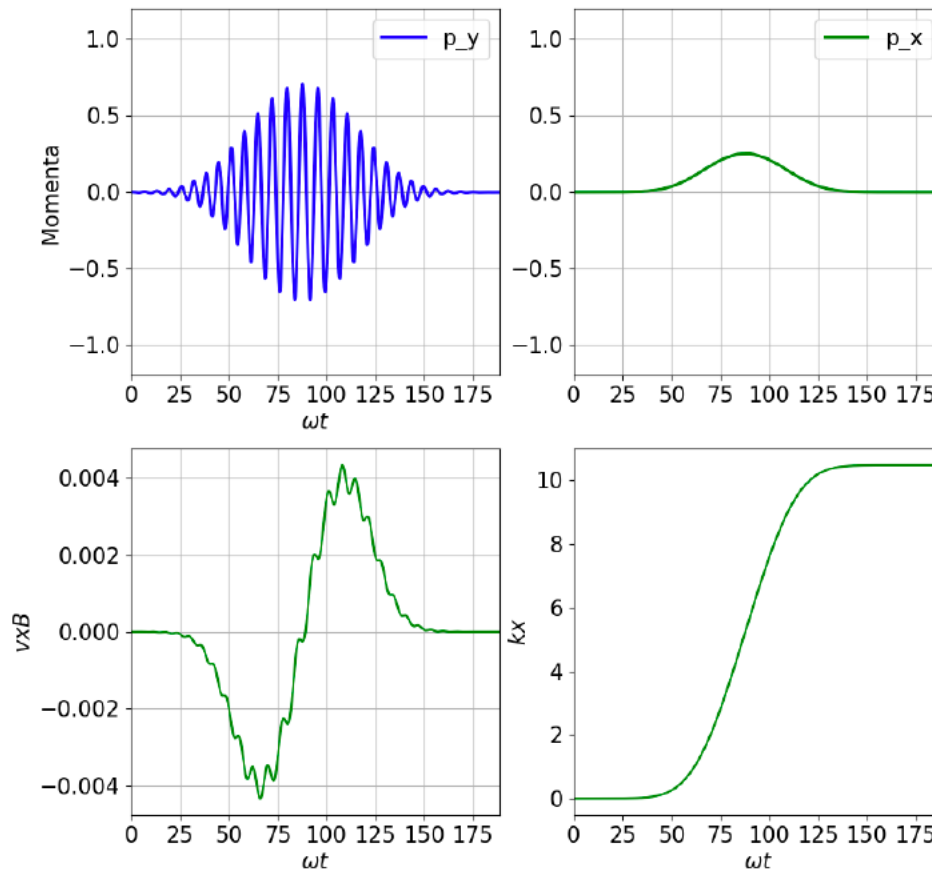
Pulse with  
*temporal envelope*  
in the wave vector  
Eq. (16).

$$\mathbf{A}(x, t) = f(t) a_0 \cos \phi,$$

No net energy  
gain!  
Lawson-Woodward  
theorem

Courtesy of Paul Gibbon, CAS 2019  
Plasma acceleration

## Finite pulse duration - CP Circular Polarization



No oscillations in  $p_x$ , but drift still there.

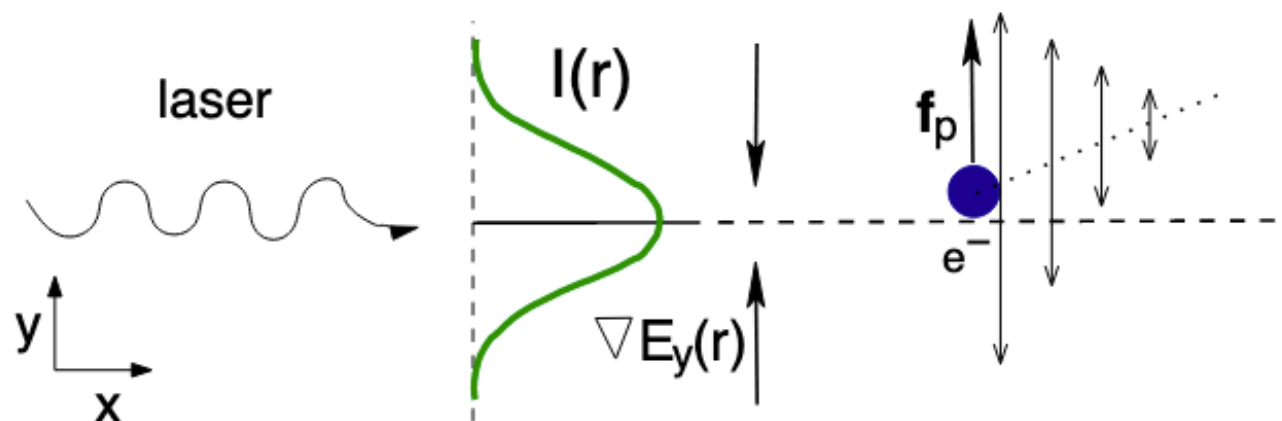
$v \times B$  oscillations also nearly vanish, but 'DC' part retained:

**longitudinal ponderomotive force!**

*Courtesy of Paul Gibbon, CAS 2019*  
Plasma acceleration

# Motion in laser focus

- Single electron oscillating slightly off-centre of focused laser beam:



- After 1st quarter-cycle, sees **lower** field
  - Doesn't quite return to initial position
- ⇒ Accelerated away from axis

*Courtesy of Paul Gibbon, CAS 2019*

## Ponderomotive force: transverse

In the limit  $v/c \ll 1$ , the equation of motion (25) for the electron becomes:

$$\frac{\partial v_y}{\partial t} = -\frac{e}{m} E_y(\mathbf{r}). \quad (21)$$

Taylor expanding electric field about the current electron position:

$$E_y(\mathbf{r}) \simeq E_0(y) \cos \phi + y \frac{\partial E_0(y)}{\partial y} \cos \phi + \dots,$$

where  $\phi = \omega t - kx$  as before.

To lowest order, we therefore have

$$v_y^{(1)} = -v_{\text{os}} \sin \phi; \quad y^{(1)} = \frac{v_{\text{os}}}{\omega} \cos \phi,$$

where  $v_{\text{os}} = eE_L/m\omega$ .

*Courtesy of Paul Gibbon, CAS 2019*

## Ponderomotive force: transverse (contd.)

Substituting back into Eq. (21) gives

$$\frac{\partial v_y^{(2)}}{\partial t} = -\frac{e^2}{m^2 \omega^2} E_0 \frac{\partial E_0(y)}{\partial y} \cos^2 \phi.$$

Multiplying by  $m$  and taking the laser cycle-average,

$$\bar{f} = \int_0^{2\pi} f \, d\phi,$$

yields the **transverse** ponderomotive force on the electron:

$$f_{py} \equiv m \overline{\frac{\partial v_y^{(2)}}{\partial t}} = -\frac{e^2}{4m\omega^2} \frac{\partial E_0^2}{\partial y}. \quad (22)$$

*Courtesy of Paul Gibbon, CAS 2019*

# Ponderomotive force

## Ponderomotive force

While the direct derivation of the relativistic ponderomotive force is quite involving, we can use identification of the ponderomotive potential as the mean kinetic energy of the quivering electrons as a short-cut:

$$\bar{E}_{kin} = \Phi_{pond} = -m_e c^2 \langle \gamma - 1 \rangle \stackrel{\gamma = \sqrt{1 + \frac{a_0^2}{2}}}{\propto} \sqrt{I}$$

This yields the relativistic ponderomotive force as:

$$\vec{F}_{pond} = -m_e c^2 \nabla \langle \gamma \rangle = -\frac{m_e c^2}{e} \nabla \sqrt{\frac{a_0^2}{2}}$$

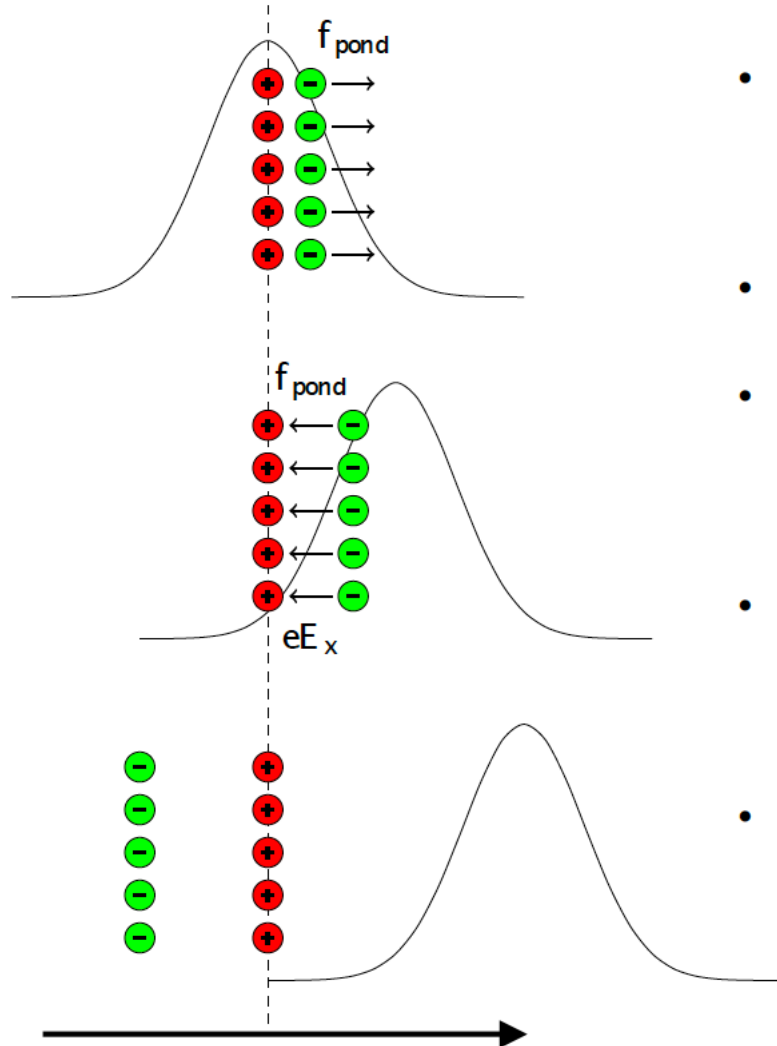
	non-relativistic	relativistic
$F_{pond}$	$-\frac{e^2}{4m_e \omega_L^2} \nabla(E_L^2)$	$-\frac{mc^2}{e} \nabla \sqrt{a_0^2/2}$
$\Phi_{pond}$	$\frac{e^2}{4m_e \omega_L^2} E_L^2$	$\frac{mc^2}{e} \langle \gamma - 1 \rangle$
proportionality	$I, \nabla I$	$\sqrt{I}, \nabla \sqrt{I}$

*Courtesy of Stephan Karsch, CAS 2019*  
Plasma acceleration



# Ponderomotive force

## Double ponderomotive push

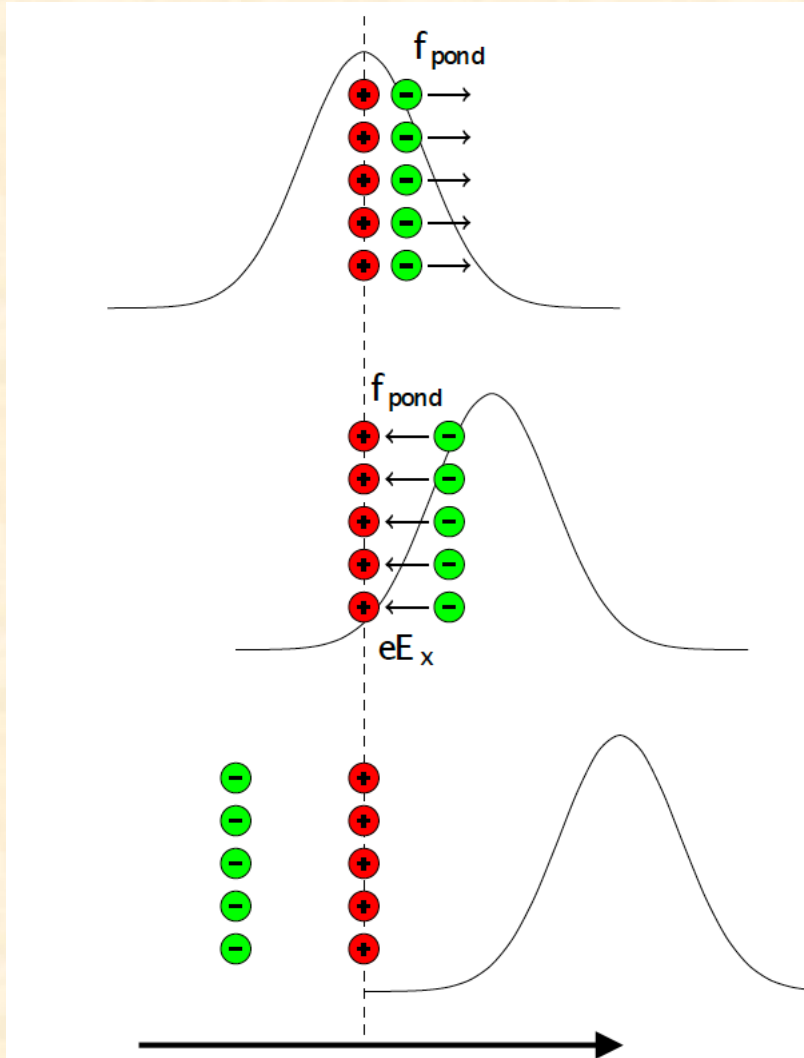


- Two kicks by the ponderomotive force, corresponding to the rising and the falling edge of the laser pulse.
- Optimum pulse duration  $\tau_{FWHM} = 0.37 \lambda_p / c$ .
- Wake excitation is dominated by the rising edge kick due to longer interaction between co-moving electrons and driver.
- Resulting charge separation separation causes electric fields to exhibit a strong longitudinal component.
- The wave structure travels with  $v_{ph} = c\eta$ , and hence can constantly accelerate a co-moving electron.

*Courtesy of Stephan Karsch, CAS 2019*  
Plasma acceleration

# Quizz:

## Increasing the pressure

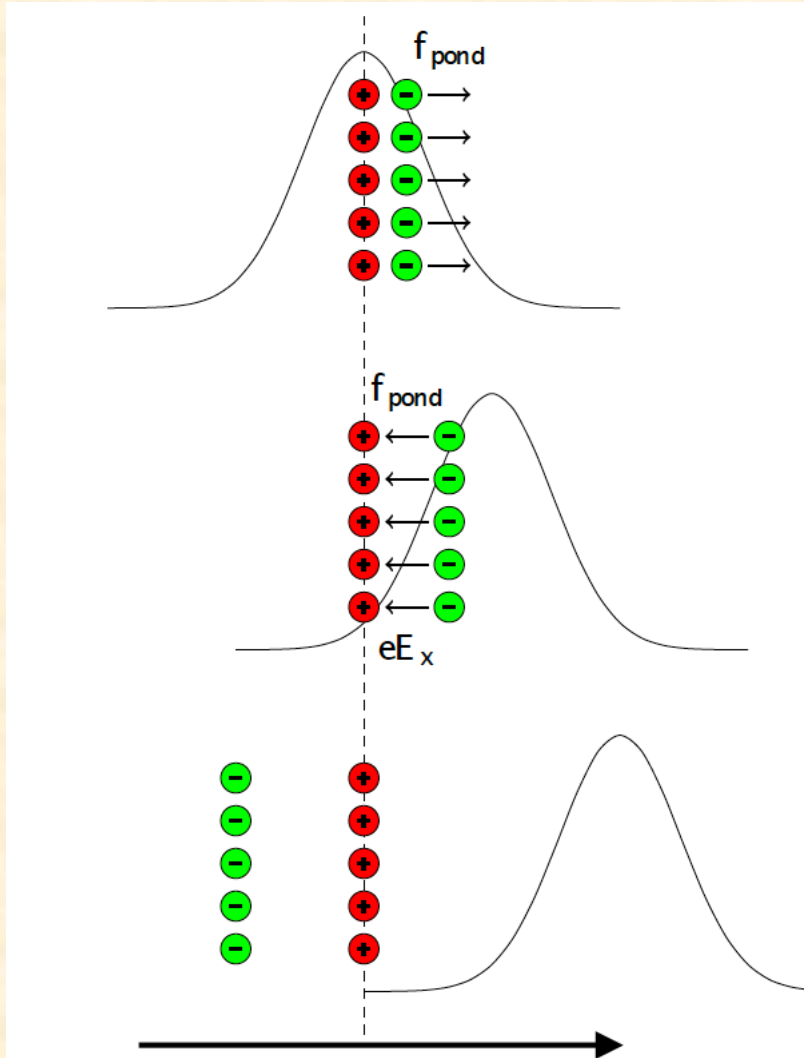


What happens if the gas pressure is increased?

- (a) Nothing
- (b) The number of electrons creating the field will be higher and hence there will be a higher accelerating gradient
- (c) Proportionally fewer electrons will be displaced and hence the gradient will be lower.

# Quizz:

## Answer (b)



The number of electrons creating the field will be higher and hence there will be a higher accelerating gradient

# Linear wakefield

## Linear wakefields

For small laser intensities ( $a_0 \ll 1$ ), the plasma density is only weakly perturbed  $\delta n_e \ll n_{e,0}$  and the continuity equation can be written as:

$$\frac{\partial \delta n_e}{\partial t} + n_{e,0} \nabla \vec{v} = 0$$

The above expression and Poisson's equation can be now inserted into the derivative of the Lorentz force. Keeping in mind  $\nabla A = 0$  (Coulomb gauge) and  $\mathbf{p} = m_e \mathbf{v}$  yields for initially resting electrons at low intensities, i.e.,  $\gamma = 1 + a^2/2$ :

$$\left( \frac{\partial^2}{\partial t^2} + \omega_p^2 \right) \frac{\delta n_e}{n_{e,0}} = c^2 \nabla^2 \frac{a^2}{2}$$

The RHS represents the driving term of a forced oscillator, and is proportional to the ponderomotive force  $F_{\text{pond}} = m_e c^2 \nabla^2 a^2/2$ . With Poisson's equation we express the charge imbalance with the scalar wake potential in the moving frame coordinates ( $\xi = z - v_g t$ ,  $\tau = t$ )

$$\left( \frac{\partial^2}{\partial \xi^2} + k_p^2 \right) \phi = k_p^2 \frac{a^2}{2}$$

Assuming a radial symmetry, an analytical solution of the inhomogeneous wave equation can be found in 3D. It is given by

$$\phi(r, \xi) = -\frac{k_p}{4} \int_{\xi}^{\infty} a^2(r, \xi') \sin(k_p (\xi - \xi')) d\xi'$$

*Courtesy of Stephan Karsch, CAS 2019*

Plasma acceleration

# Linear wakefield

## Linear wakefields II

For a Gaussian laser envelope  $a = a_0 \exp(-\xi^2/(c\tau_0)^2) \exp(-r^2/w_0^2)$ , the solution of the integral for  $\xi \rightarrow -\infty$ , i.e. after the laser transit is given by:

$$\phi(r, \xi) = -a_0^2 \sqrt{\frac{\pi}{2}} \frac{k_p}{4} c\tau_0 e^{-(2r^2/w_0^2)} e^{-(k_p c\tau_0)^2/8} \sin(k_p \xi)$$

From this scalar potential  $\phi$  the electric field and the electron density can be derived as:

$$\frac{E_z}{E_{p,0}} = -\frac{1}{k_p} \frac{\partial \phi}{\partial \xi}, \quad \frac{E_r}{E_{p,0}} = -\frac{1}{k_p} \frac{\partial \phi}{\partial r}, \quad \frac{\delta n_e}{n_{e,0}} = -\frac{1}{k_p^2} \frac{\partial^2 \phi}{\partial \xi^2},$$

$E_{p,0}$  corresponds to the maximal electric field of the plasma wave in the linear regime, known as the cold fluid wavebreaking field:

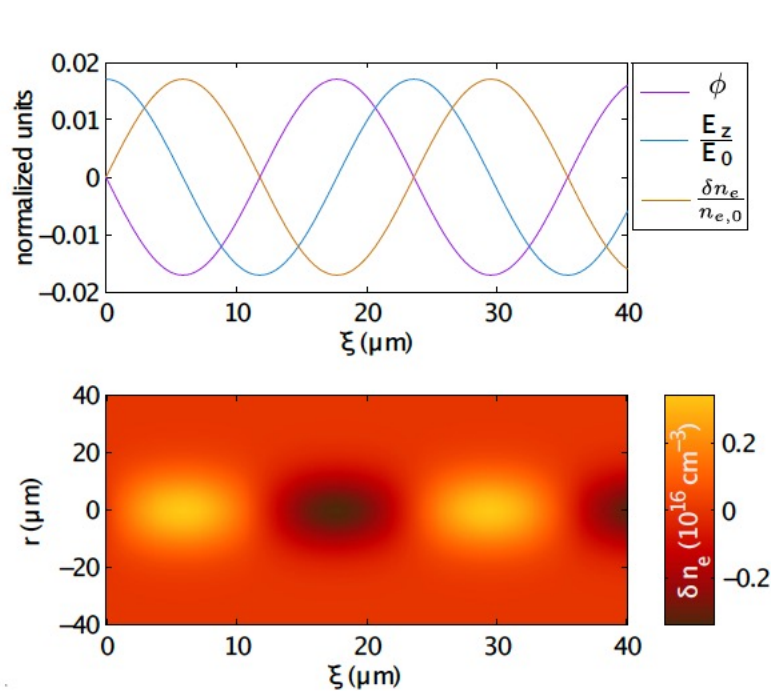
$$E_{p,0} = \frac{m_e c \omega_p}{e}, \quad E_{p,0} [\text{GV/m}] = 96 \sqrt{n_{e,0} [10^{18} \text{cm}^{-3}]}$$

*Courtesy of Stephan Karsch, CAS 2019*

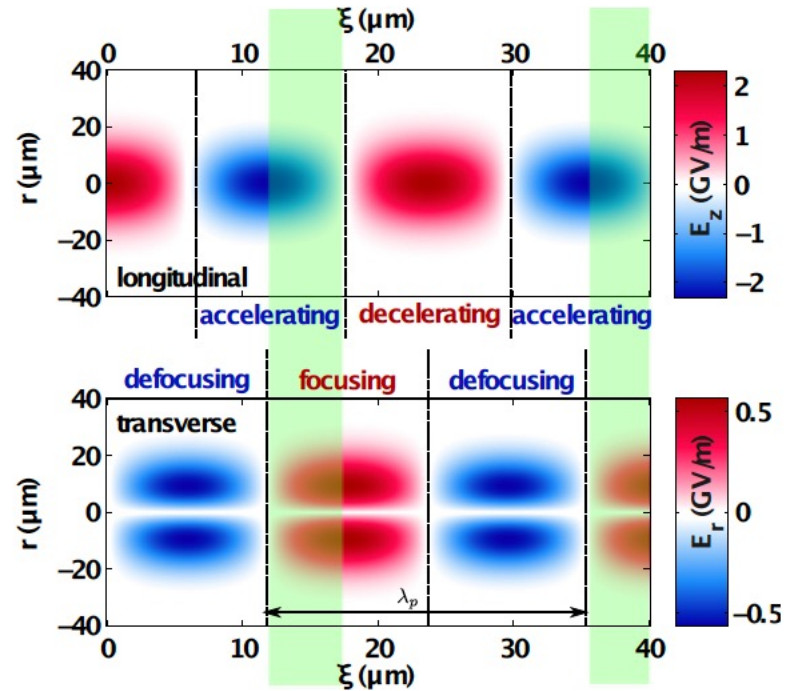
Plasma acceleration

# Linear wakefield

## Linear wakefields III



Top: Normalized plasma potential  $\phi$ , longitudinal electric field  $E_z/E_0$  and density perturbation  $\delta n_e/n_{e,0}$  on axis ( $r = 0$ ). Bottom: color coded plasma density perturbation  $\delta n_e(r, \xi)/n_{e,0}$  generated by the ponderomotive force in the vicinity of a Gaussian laser focus.



top: Spatial extent of the longitudinal  $E_z(r, \xi)$  and bottom: the radial electric field  $E_r(r, \xi)$ . The green area marks a  $\lambda_p / 4$ -phase region of the wakefield with an accelerating and transverse focusing field.

3D linear wakefield quantities in the co-moving frame created by a laser pulse with  $a_0 = 0.2$ ,  $t_{\text{FWHM}} = 28 \text{ fs}$  and  $d_{\text{FWHM}} = 22 \mu\text{m}$  in a plasma density of  $2 \times 10^{18} \text{ cm}^{-3}$

*Courtesy of Stephan Karsch, CAS 2019*

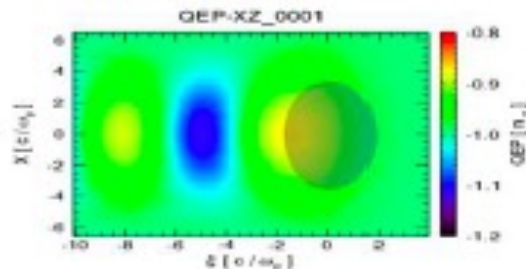
Plasma acceleration



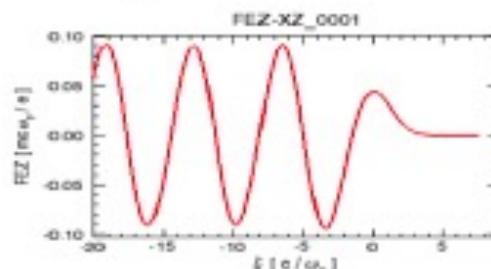
# From linear to non-linear

## From Linear to Non-Linear

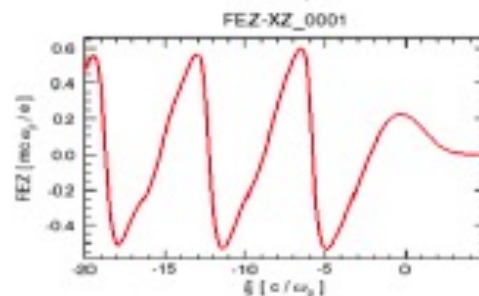
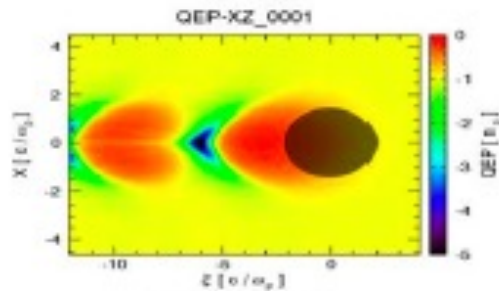
Electron density :



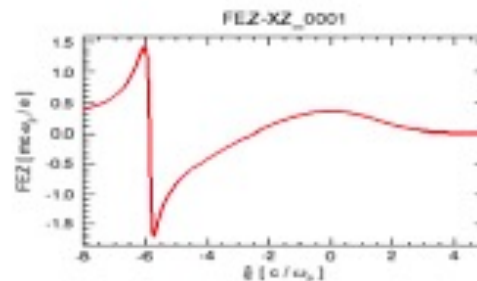
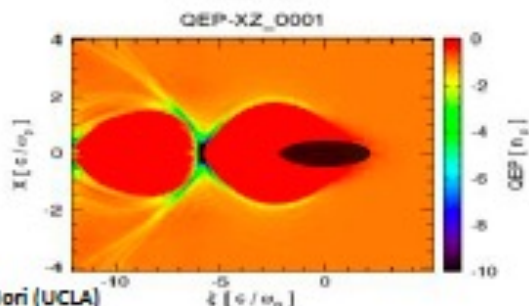
Longitudinal fields :



$n_b \ll n_{pe}$  – linear regime



$n_b \sim n_{pe}$  – non-linear wakes



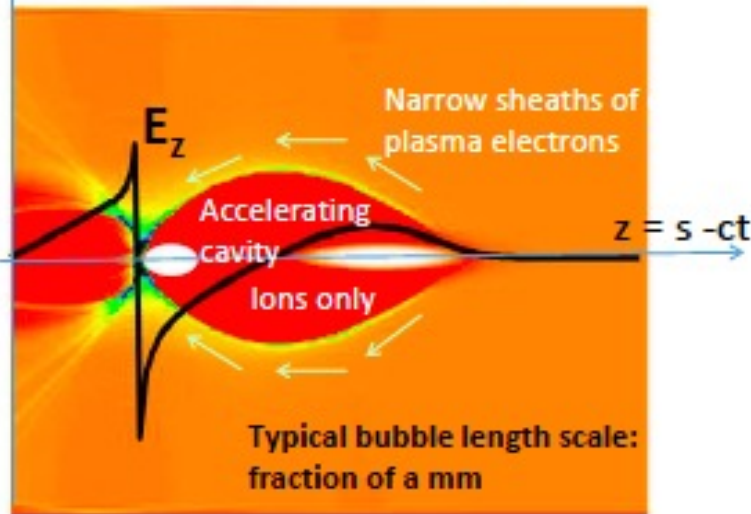
$n_b \gg n_{pe}$  – blow-out regime

W. Mori (UCLA)

Courtesy of Edda Gschwendtner, CAS 2019  
Plasma acceleration

# Blow-out regime

## Blow-out Regime

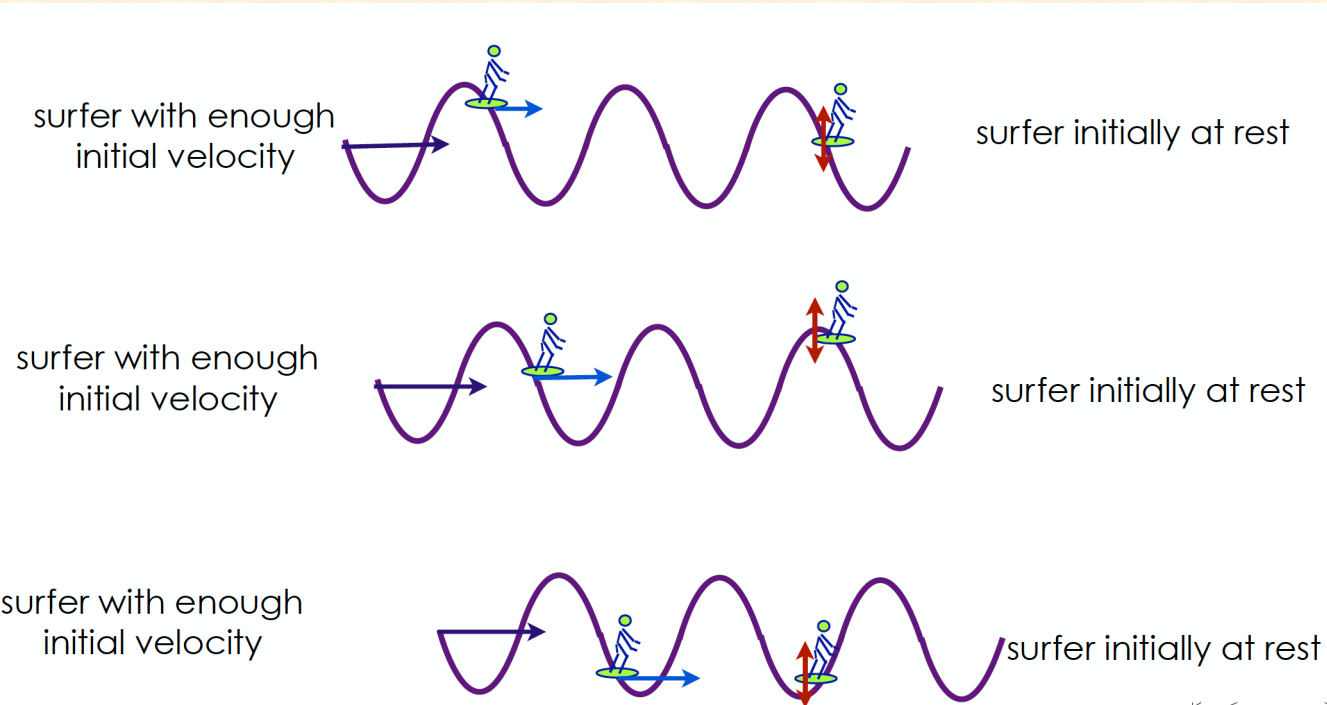


- Space-charge force of the driver blows away all the plasma electrons in its path, leaving a uniform layer of ions behind (ions move on a slower time scale).
- Plasma electrons form a narrow sheath around the evacuated area, and are pulled back by the ion-channel after the drive beam has passed
- An accelerating cavity is formed in the plasma
- The back of the blown-out region: ideal for electron acceleration

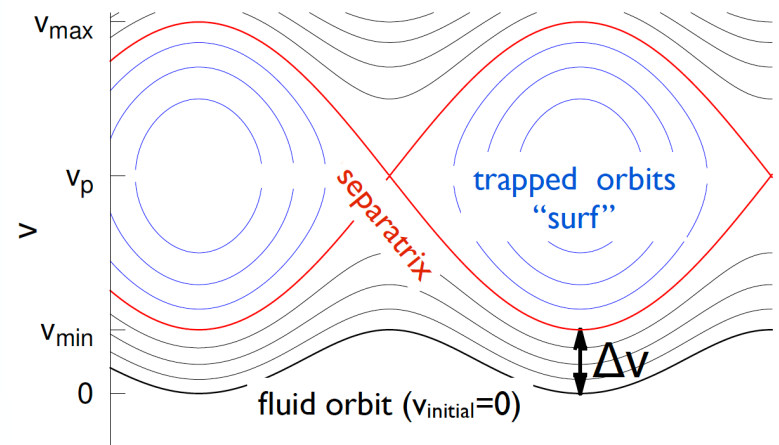
- High charge witness acceleration possible → charge ratio to witness of same order
- Linear focusing in  $r$ , for electrons; very strong quadrupole (MT/m)
- High transformer ratios ( $>2$ ) can be achieved by shaping the drive bunch
- $E_r$  independent of  $x$ , can preserve incoming emittance of witness beam



# Surfing the wave



Courtesy of V. Malka



# Trapping condition

## Trapping condition

When can an electron be trapped in the plasma wave?

Consider Hamiltonian of an electron interacting with the laser field in the presence of a plasma wave (normalized quantities):

$$H(z, u_z) = \underbrace{\sqrt{1 + u_{\perp}^2 + u_z^2}}_{=\gamma} - \phi(z - v_g t)$$

For an initially resting electron, due to conservation of canonical momentum,  $u_{\perp} = a$ . The second term represents the wave's potential. The time dependence can be eliminated by a canonical transformation  $(z, u_z) \rightarrow (\xi, u_z)$ <sup>1</sup>. The time-independent Hamiltonian then reads:

$$H(z, u_z) = \sqrt{1 + a(\xi)^2 + u_z(\xi)^2} + \phi(\xi) - \beta_g u_z(\xi)$$

$H(\xi, u_z) = H_0 = \text{const.}$  describes the motion of an electron with an initial energy  $E = H_0$  on a distinct orbit in the plasma wave. Solving the expression for the Hamiltonian for  $u_z(\xi)$  gives the trajectory of the electron in the longitudinal phase space  $(\xi, u_z)$ :

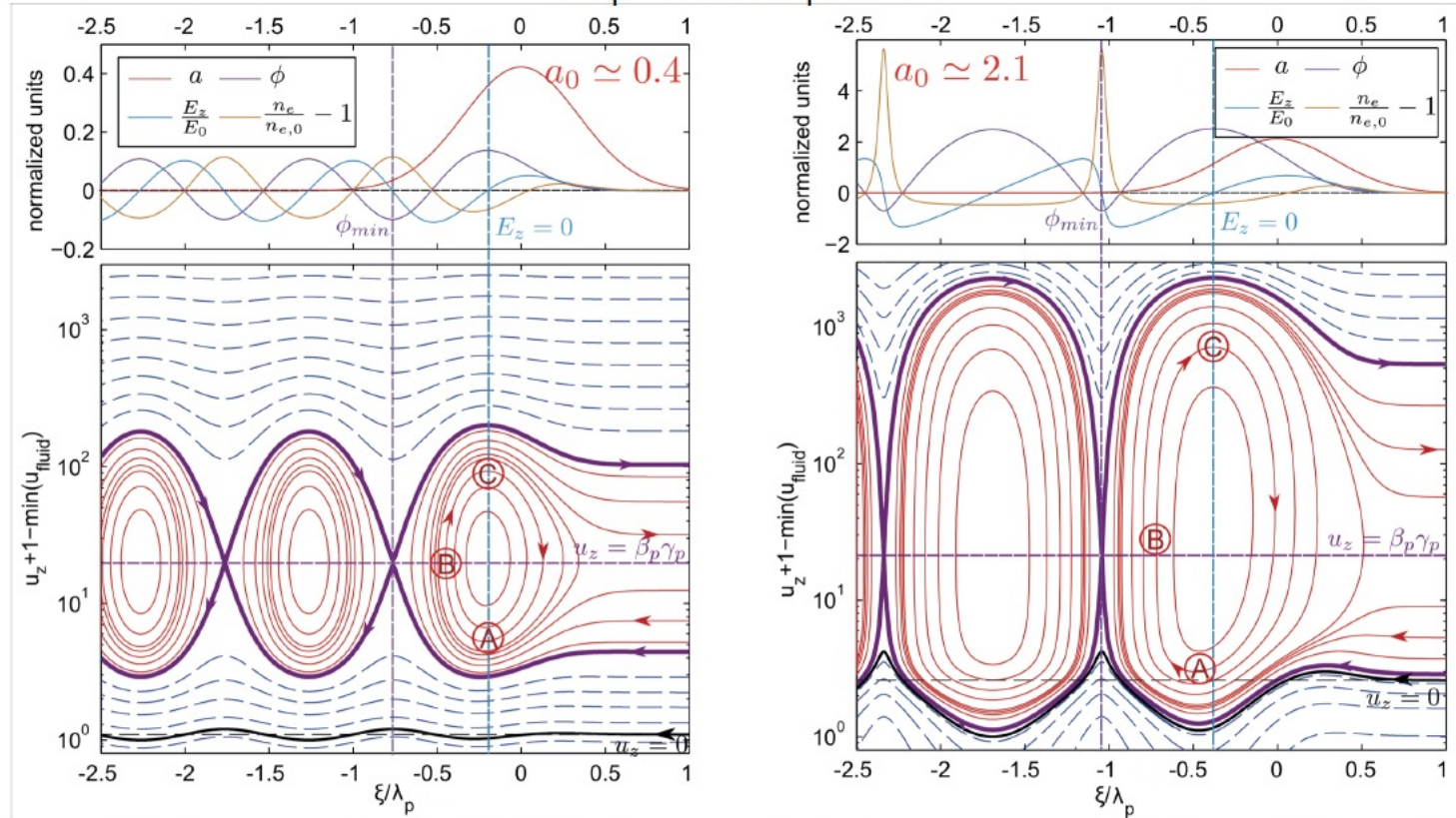
$$u_z = \beta_g \gamma_g^2 (H_0 + \phi) \pm \gamma_g \sqrt{\gamma_g^2 (H_0 + \phi)^2 - \gamma_{\perp}^2}$$

$u_z(\xi)$  represents an electron orbit of constant total energy for a given set of  $a(\xi)$ ,  $\phi(\xi)$  and  $H_0$

<sup>1</sup>With a generating function  $F(z, u_z) = u_z \times (z - v_g t)$  the new Hamiltonian reads  $H = H' - 1/c \frac{\partial F}{\partial t}$

# Trapping condition

## Separatrix plots



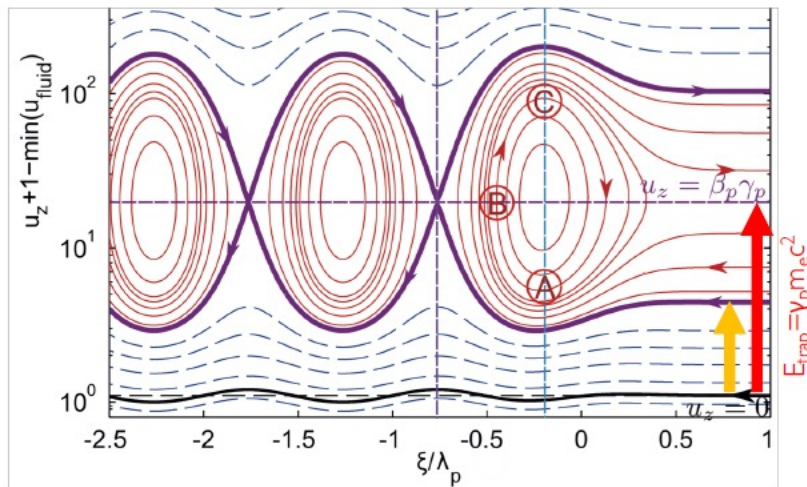
(red): trapped electrons on closed orbit. (blue): untrapped electrons on open orbit. (purple) Separatrix separating open and closed orbits with a radicand equal to zero. It crosses itself at  $\phi = \min$  (purple vertical line). The Hamiltonian of the separatrix is given by  $H_{\text{sep}} = v_{\perp}(\xi_{\min})/v_g - \phi_{\min}$ . Electrons initially at rest ( $H_{\text{fluid}} = 1, u_{\perp}(\xi = +\infty) = u_z(\xi = +\infty) = 0$ , black) do not gain momentum from the plasma wave. Electrons with a too low/high initial momentum (dashed blue lines)  $|H_0| > |H_{\text{sep}}|$  are moving on open orbits

*Courtesy of Stefan Karsch, CAS 2019*



# Trapping condition

Trapping condition for  $e^-$  overtaken by wakefield (external injection)



In 1-D, the trapping condition reads:

$$E_{\text{trap}} = m_e c^2 \left( \sqrt{1 + u_{z,\text{sep}}^2(+\infty)} - 1 \right)$$

with:

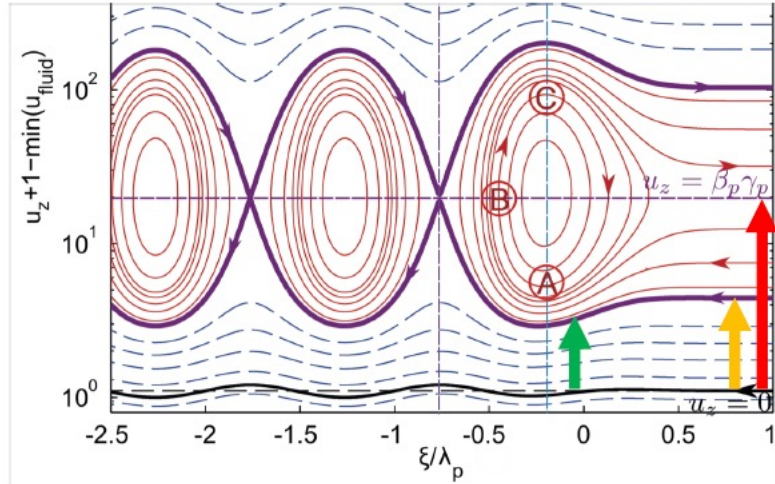
$$u_{z,\text{sep}}(+\infty) = \beta_p \gamma_p^2 H_{\text{sep}} - \gamma_p \sqrt{\gamma_p^2 H_{\text{sep}}^2 - 1}$$

being the separatrix distance in front of the laser ( $a_0 = \phi = u_{\perp} = 0$ )

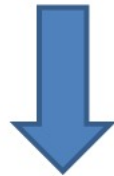
- Electrons with a forward momentum substantially lower (how much depends on wake amplitude) can be caught and gain maximum energy at point C if acceleration would terminate there.

# Trapping condition

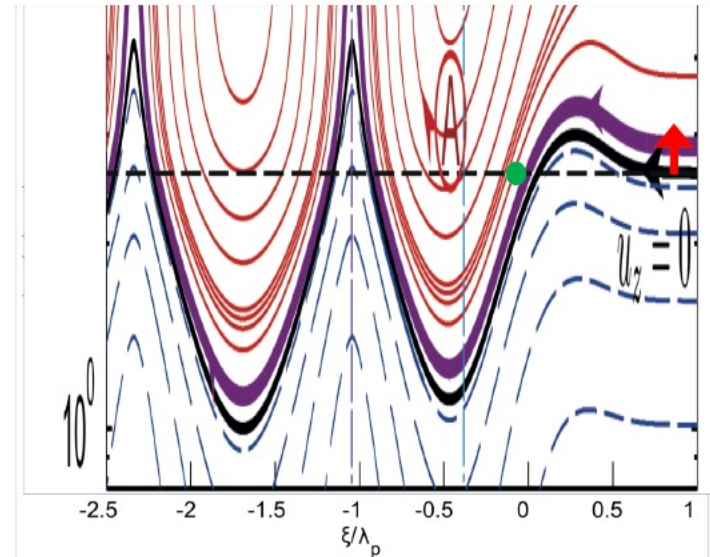
How about even lower thresholds?



Electrons gain threshold energy inside wake bucket



Colliding pulse injection



Electrons are born inside wake bucket



Ionization injection

*Courtesy of Stefan Karsch, CAS 2019*  
Plasma acceleration

# Trapping condition

## Colliding pulse (beat wave) injection

Consider two counter-propagating, c.p. laser pulses:

$$a_{1/2} = \frac{a_{1/2}(t)}{\sqrt{2}} \left( \cos(k_L z \pm \omega_L t) \vec{e}_x + \sin(k_L z \pm \omega_L t) \vec{e}_y \right)$$

where  $a_{0,1/2}(t)$  are the temporal pulse shapes for both pulses

With the beat-wave Hamiltonian

$$H_{beat} = \sqrt{1 + u_{\perp}^2 + u_z^2} = \sqrt{1 + (a_1 + a_2)^2 + u_z^2}$$

we get a beat-wave separatrix:

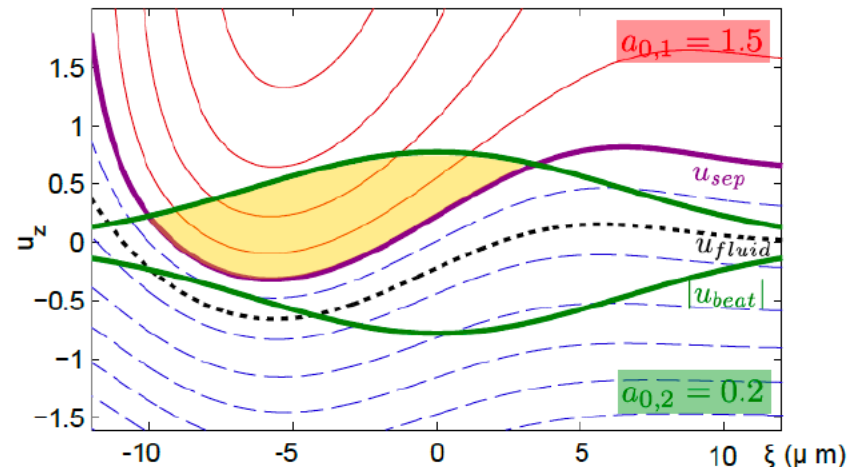
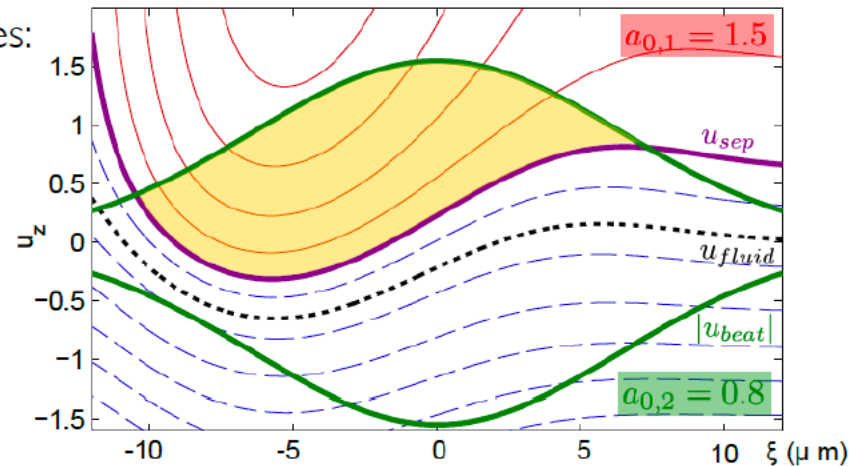
$$u_{beat}(t) = \pm \sqrt{a_{0,1}(t) a_{0,2}(t) (1 - \cos(2\omega_L t))}$$

$$u_{beat,max/min}(t) = \pm \sqrt{2a_{0,1}(t) a_{0,2}(t)}$$

$$W_{beat}(t) = m_e c^2 \sqrt{1 + u_{beat}(t)^2} - 1$$

Injection if (in co-moving frame):

$$u_{beat,max}(\xi) > u_{sep}(\xi)$$



Courtesy of Stefan Karsch, CAS 2019

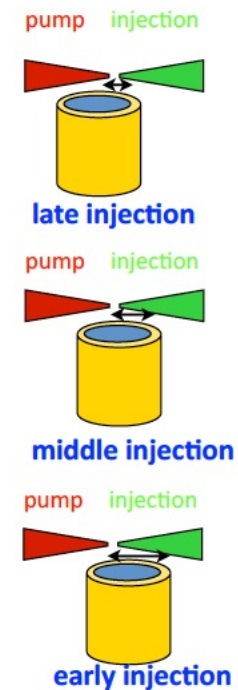
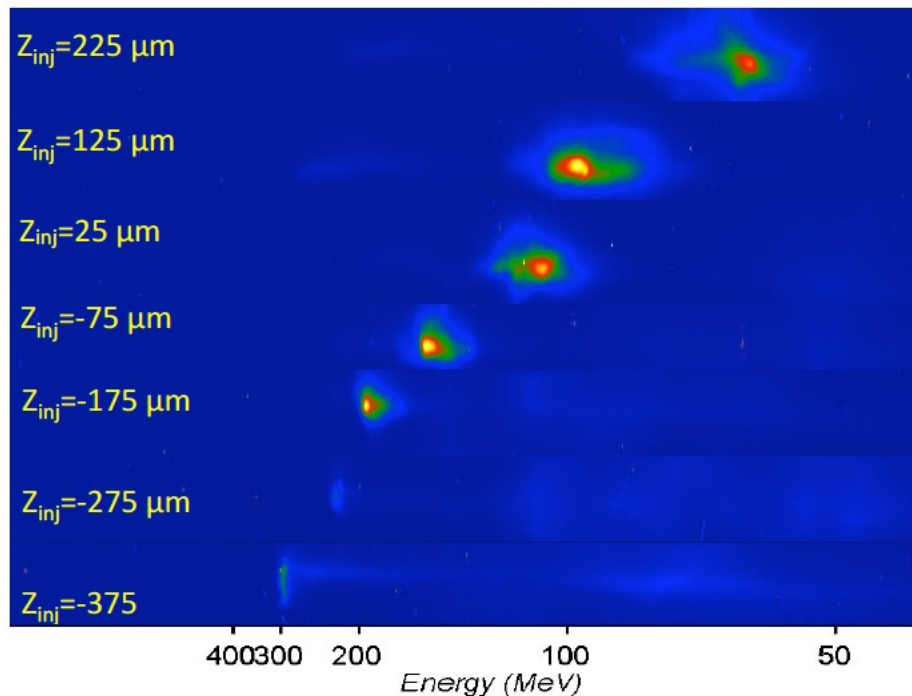
Plasma acceleration



# Trapping condition

Colliding pulse (beat wave) injection exp.

- Localized injection leading to quasi-monochromatic beams
- Adjustable energy via tuning of collision (injection) position



accelerating distance  $\longleftrightarrow$

J Faure et al., Nature 444, 737 (2006)

*Courtesy of Stejan Karsch, CAS 2019*  
Plasma acceleration

# Trapping condition

## Ionization injection

Gas target contains traces of high- $Z$  gas, which is ionized by the peak of the laser and born at  $\xi_{\text{ion}} \sim 0$  at rest ( $u_z(\xi_{\text{ion}}) \sim 0$ ):

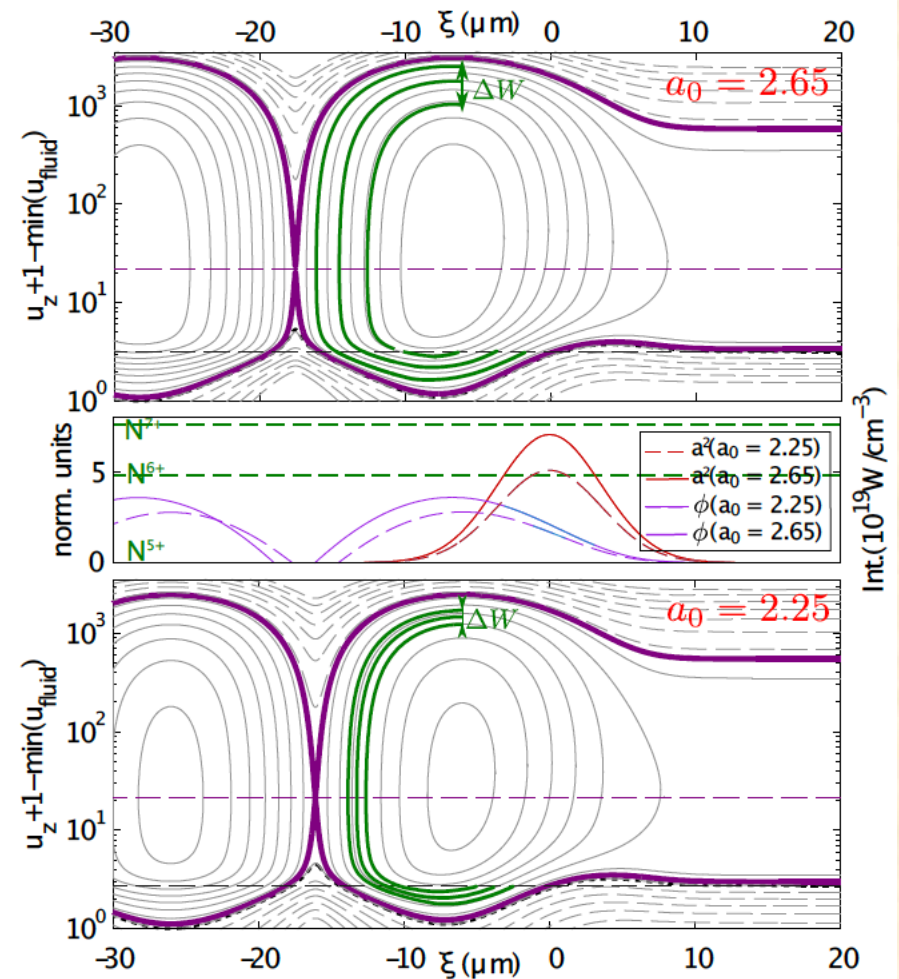
$$H_{\text{ion}} = 1 - \phi(\xi_{\text{ion}})$$

Trapping condition<sup>1</sup> for sin-envelope pulses:

$$1 - \gamma_p^{-1} \leq \phi(\xi_{\text{ion}}) - \phi_{\text{min}} \leq \phi_{\text{max}} - \phi_{\text{min}} \sim \underbrace{\left( \frac{\pi}{8} + \frac{1}{4} \right)}_{\sim 0.64} a_0^2$$

Ionization injection only works for relativistic intensity ( $a_0^2 > 1.6$ ) pulses!

(even if ionization threshold would be lower)

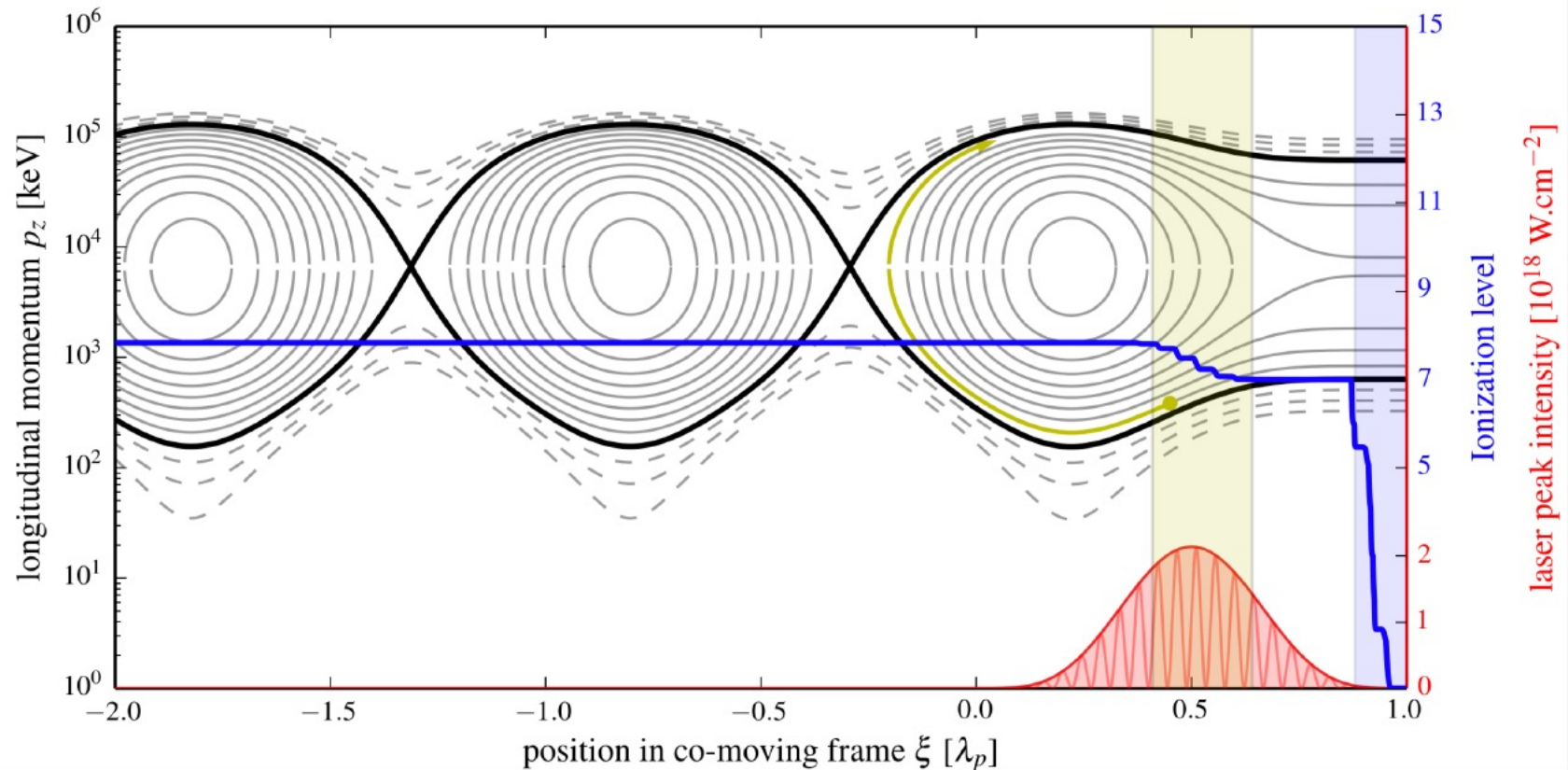


<sup>1</sup>Chen et al, Phys. Plasmas 19,033101 (2012)

# Trapping condition

Ionization injection II

Oxygen trace gas

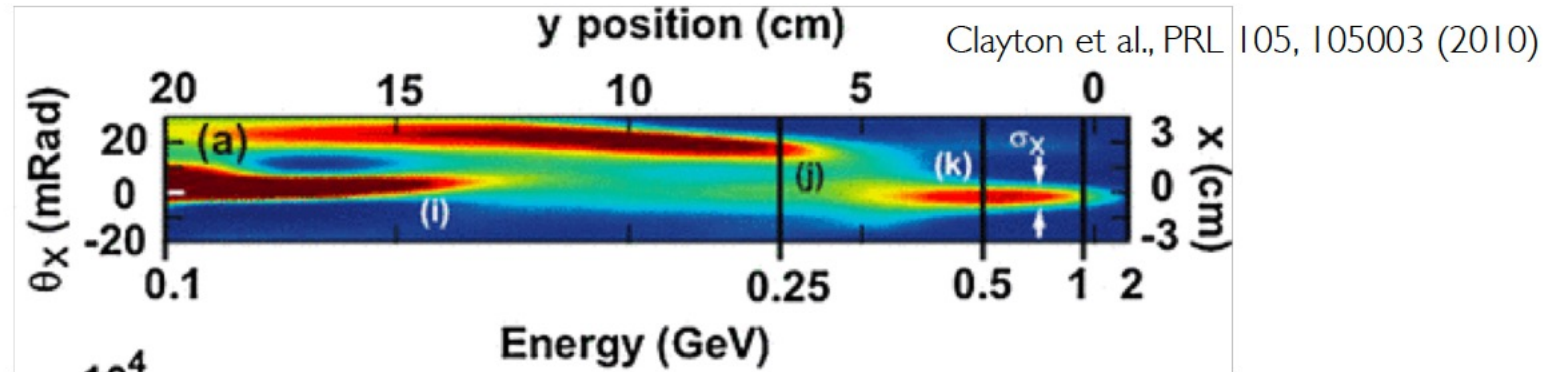


Courtesy of Stefan Karsch, CAS 2019  
Plasma acceleration

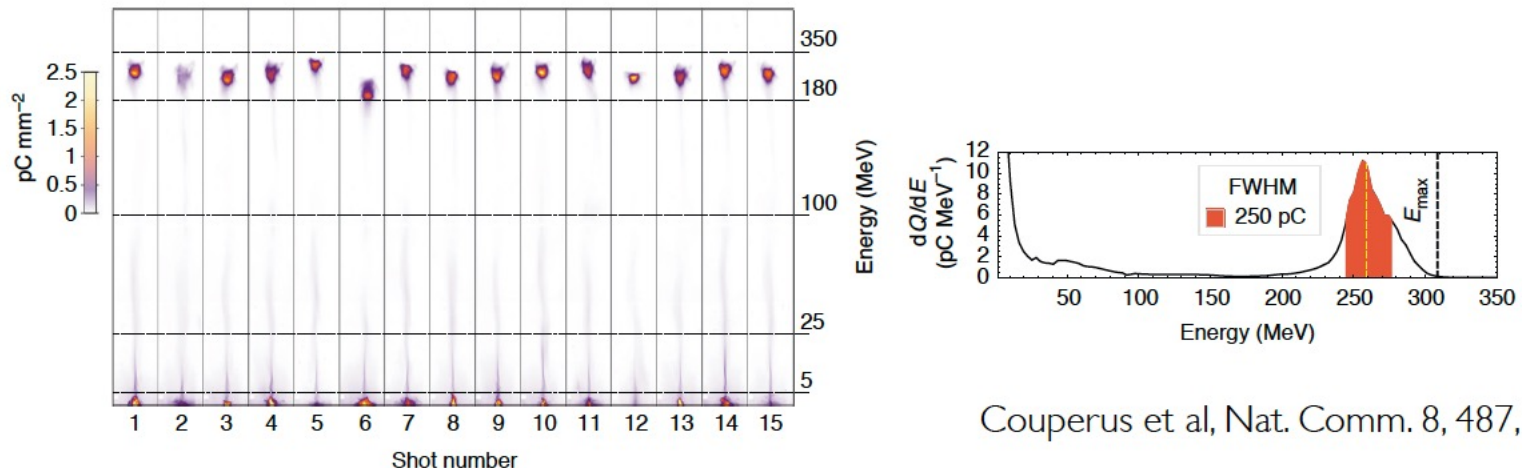
# Trapping condition

Ionization injection exp.

- Constant injection commonly leads to broadband spectra, but high charge...



- ... which can be used to fully beamload and truncate the injection



Couperus et al, Nat. Comm. 8, 487, 2017

*Courtesy of Stefan Karsch, CAS 2019*

Plasma acceleration



# Trapping condition

„Longitudinal injection“

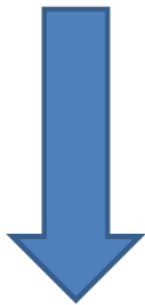
Instead of giving an electron the correct energy at the correct phase, it is possible to shift the wake phase to gobble up electrons from other phase positions.

Any sudden shift in plasma wavelength our driving phase will shift the wake phase.

Shift by laser intensity variation

Shift by density step / slope

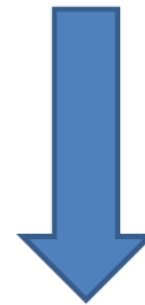
Shift by driver swap



longitudinal/transverse  
self-injection



density  
down ramp/shock front  
injection



Hybrid  
injection

all these schemes will cause the wave to break momentarily or continuously

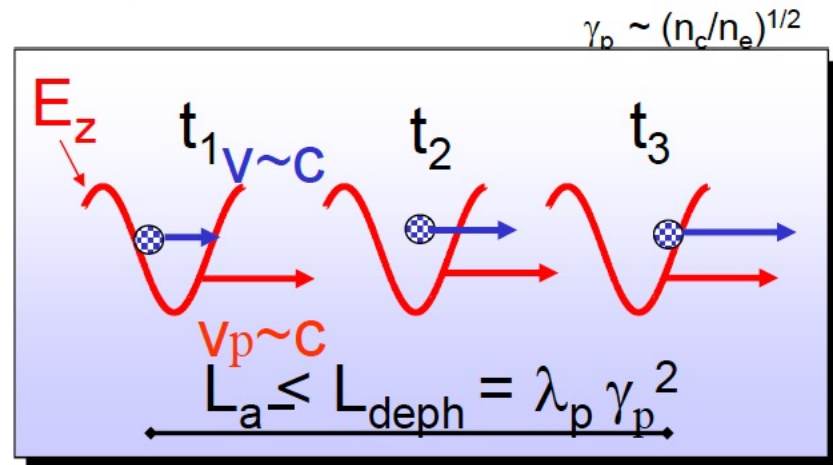
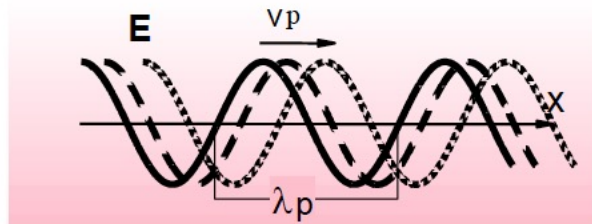
*Courtesy of Stefan Karsch, CAS 2019*

Plasma acceleration

# Dephasing length

Relativistic electrons are trapped and accelerated over the dephasing length

- ➡ Relativistic plasma wave:  
Too slow or too fast electrons do not stay long with the wave



B. Cros, CAS HGWA Sesimbra, March 2019

Courtesy of Brigitte Cros, CAS 2019

Plasma acceleration



# Dephasing length

## Energy gain over dephasing length



➡ Energy gain

$$\Delta W = e E_z L_a \sim 4mc^2 \gamma_p^2$$

➡ Relativistic factor

$$\gamma_p \sim (n_c/n_e)^{1/2}$$

$$\gamma_p = \lambda_p / \lambda_0$$

$$L_a < L_{\text{deph}} = \lambda_p \gamma_p^2$$



$n_e$	$10^{17} \text{cm}^{-3}$	$10^{19} \text{cm}^{-3}$
$\gamma_p$	100	10
$L_a$	1 m	1 mm
$\Delta W_{\text{max}}$	20 GeV	200 MeV

B. Cros, CAS HGWA Sesimbra, March 2019

14



*Courtesy of Brigitte Cros, CAS 2019*  
Plasma acceleration

Some simple equations

# Some simple equations about laser-plasma acceleration

- Plasma:

$$n_e [cm^{-3}] = 2,429 \times 10^{16} \times Z \times p [mbar]$$

$$n_e [10^{17} cm^{-3}] = 0,486 p [mbar]. \quad \text{For H}_2 \text{ or He}$$

$$\omega_p = \sqrt{\frac{n_e e^2}{m \epsilon_0}}$$

Plasma pulsation

$$\frac{\omega}{\omega_p} = \sqrt{\frac{n_c}{n_e}}$$

$$n_c = \frac{1,11485 \times 10^{21}}{\lambda^2 [\mu m^2]} cm^{-3}$$

Critical density.  
Typically  $n_e \ll n_c$  in  
ALP

$$\lambda_p = \lambda \times \sqrt{\frac{n_c}{n_e}}$$

# Some simple equations about laser-plasma acceleration

- Laser:

$$a_0 = 0,855 \sqrt{I [10^{18} W / cm^2] \times \lambda^2 [\mu m^2]}$$

- Acceleration:

$$E_0 = \frac{mc\omega_p}{e} = \frac{2\pi mc^2}{e} \times \frac{1}{\lambda_p}$$

$$E_0 [GV / m] = \frac{3,2107 \times 10^{12}}{\lambda_p [\mu m]}$$

- dephasingxxx

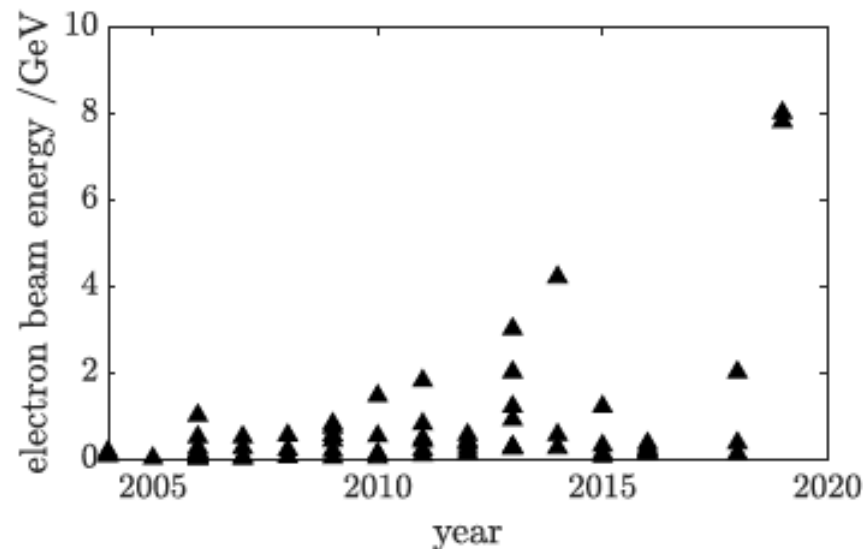
$$E_0 [GV / m] = 96,159 \sqrt{n_e [10^{18} cm^{-3}]}$$

The higher the gas pressure the better gradient but the shorter the plasma wavelength (ie the trapping volume)

Some experimental results

# Some experimental results

## Fast progress in electron beam energy



- Electron beam from laser wakefield accelerators has been going up steadily since 2004 results.

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration



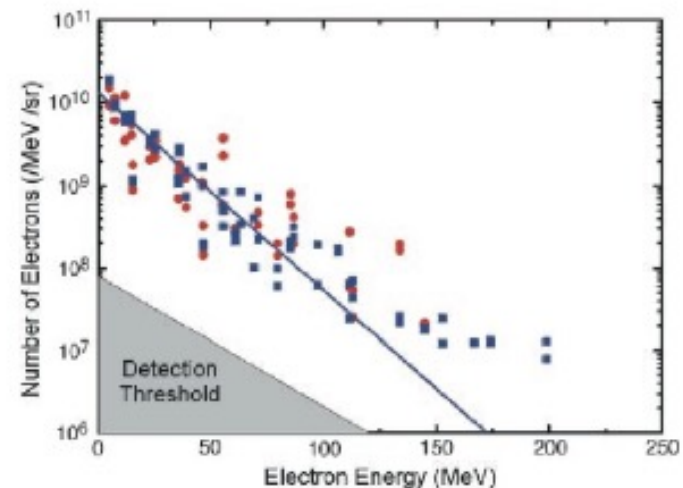
# Some experimental results

## Experiments at the energy frontier: 2002

### Electron Acceleration by a Wake Field Forced by an Intense Ultrashort Laser Pulse

V. Malka,<sup>1\*</sup> S. Fritzler,<sup>1</sup> E. Lefebvre,<sup>2</sup> M.-M. Aleonard,<sup>3</sup> F. Burgy,<sup>1</sup>  
J.-P. Chambaret,<sup>1</sup> J.-F. Chemin,<sup>3</sup> K. Krushelnick,<sup>4</sup> G. Malka,<sup>3</sup>  
S. P. D. Mangles,<sup>4</sup> Z. Najmudin,<sup>4</sup> M. Pittman,<sup>1</sup> J.-P. Rousseau,<sup>1</sup>  
J.-N. Scheurer,<sup>3</sup> B. Walton,<sup>4</sup> A. E. Dangor<sup>4</sup>

Plasmas are an attractive medium for the next generation of particle accelerators because they can support electric fields greater than several hundred gigavolts per meter. These accelerating fields are generated by relativistic plasma waves—space-charge oscillations—that can be excited when a high-intensity laser propagates through a plasma. Large currents of background electrons can then be trapped and subsequently accelerated by these relativistic waves. In the forced laser wake field regime, where the laser pulse length is of the order of the plasma wavelength, we show that a gain in maximum electron energy of up to 200 megaelectronvolts can be achieved, along with an improvement in the quality of the ultrashort electron beam.



V. Malka, *Science*, 298, 1596-1600 (2002)

- Extends to 200 MeV
- $n_e = 2.5 \times 10^{19} \text{ cm}^{-3}$ , 3 mm gas jet
- $P = 33 \text{ TW}$ , "Salle Jaune" laser at LOA

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration

# Some experimental results

## Experiments at the energy frontier: 2006

### GeV electron beams from a centimetre-scale accelerator

W. P. LEEMANS<sup>1,\*</sup>, B. NÄGLER<sup>1</sup>, A. J. GONSALVES<sup>2</sup>, Cs. TÓTH<sup>1</sup>, K. NAKAMURA<sup>1,3</sup>, C. G. R. GEDDES<sup>1</sup>, E. ESAREY<sup>1\*</sup>, C. B. SCHROEDER<sup>1</sup> AND S. M. HOOKER<sup>2</sup>

<sup>1</sup>Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, California 94720, USA

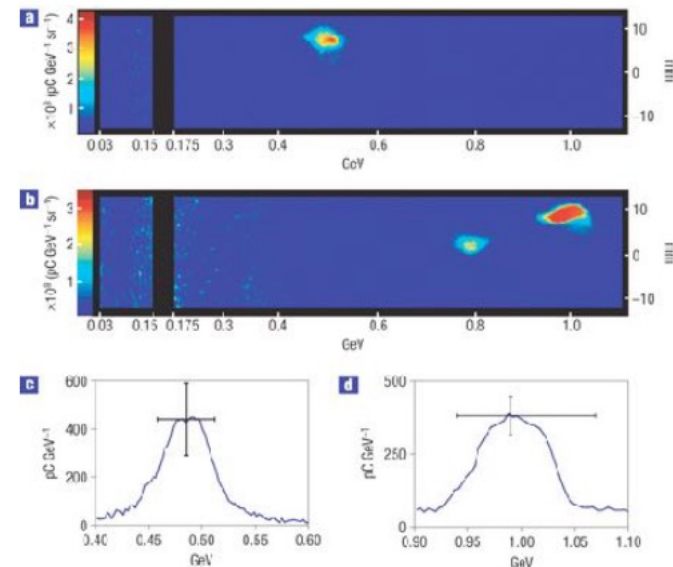
<sup>2</sup>University of Oxford, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK

<sup>3</sup>Nuclear Professional School, University of Tokyo, 22-2 Shirane-shinkai, Tokyo, Naka, Ibaraki 319-1188, Japan

\*Also at: Physics Department, University of Nevada, Reno, Nevada 89567, USA

\*e-mail: W.P.leemans@lbl.gov

W.P. Leemans, Nature Physics, 2, 696-699 (2006)



- 1.0 GeV
- $n_e = 4.3 \times 10^{18}$  cm<sup>-3</sup>, 33 mm capillary discharge waveguide
- $P = 40$  TW, TREX laser at LBNL

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration

# Some experimental results

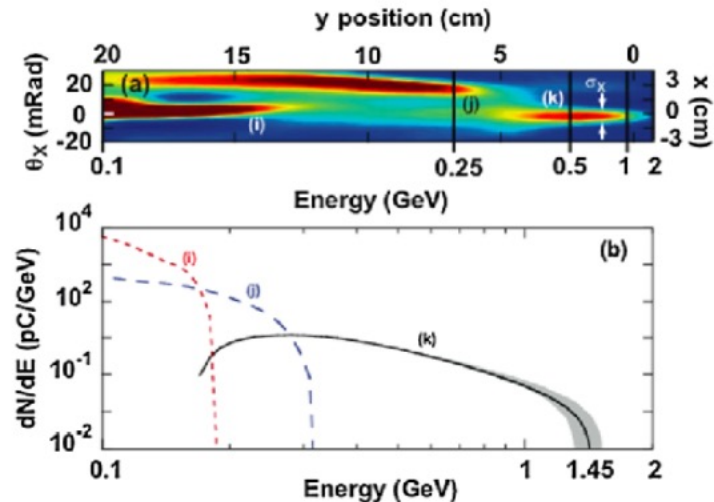
## Experiments at the energy frontier: 2010

### Self-Guided Laser Wakefield Acceleration beyond 1 GeV Using Ionization-Induced Injection

C. E. Clayton,<sup>1,\*</sup> J. E. Ralph,<sup>2</sup> F. Albert,<sup>2</sup> R. A. Fonseca,<sup>3</sup> S. H. Glenzer,<sup>2</sup> C. Joshi,<sup>1</sup> W. Lu,<sup>1</sup> K. A. Marsh,<sup>1</sup> S. F. Martins,<sup>2</sup> W. B. Mori,<sup>1</sup> A. Pak,<sup>1</sup> F. S. Tsung,<sup>1</sup> B. B. Pollock,<sup>2,4</sup> J. S. Ross,<sup>2,4</sup> L. O. Silva,<sup>2</sup> and D. H. Froula<sup>2</sup>  
<sup>1</sup>Department of Electrical Engineering, University of California, Los Angeles, California 90095, USA  
<sup>2</sup>L-399, Lawrence Livermore National Laboratory, P.O. Box 808, Livermore, California 94551, USA  
<sup>3</sup>GeL/PPPT-LA, Instituto Superior Técnico, Lisboa, Portugal  
<sup>4</sup>MAE Department, University of California, San Diego, La Jolla, California 92093, USA  
(Received 23 April 2010; published 1 September 2010)

The concepts of matched-beam, self-guided laser propagation and ionization-induced injection have been combined to accelerate electrons up to 1.45 GeV energy in a laser wakefield accelerator. From the spatial and spectral content of the laser light exiting the plasma, we infer that the 60 fs, 110 TW laser pulse is guided and excites a wake over the entire 1.3 cm length of the gas cell at densities below  $1.5 \times 10^{18} \text{ cm}^{-3}$ . High-energy electrons are observed only when small (3%) amounts of  $\text{CO}_2$  gas are added to the He gas. Computer simulations confirm that it is the  $K$ -shell electrons of oxygen that are ionized and injected into the wake and accelerated to beyond 1 GeV energy.

C. Clayton, Phys. Rev. Lett, 105, 105003 (2010)



- Extends to 1.45 GeV
- $n_e = 4.3 \times 10^{18} \text{ cm}^{-3}$ , 1.3 cm gas cell
- P = 220 TW Callisto Laser at LLNL

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration

# Some experimental results

## Experiments at the energy frontier: 2013

### ARTICLE

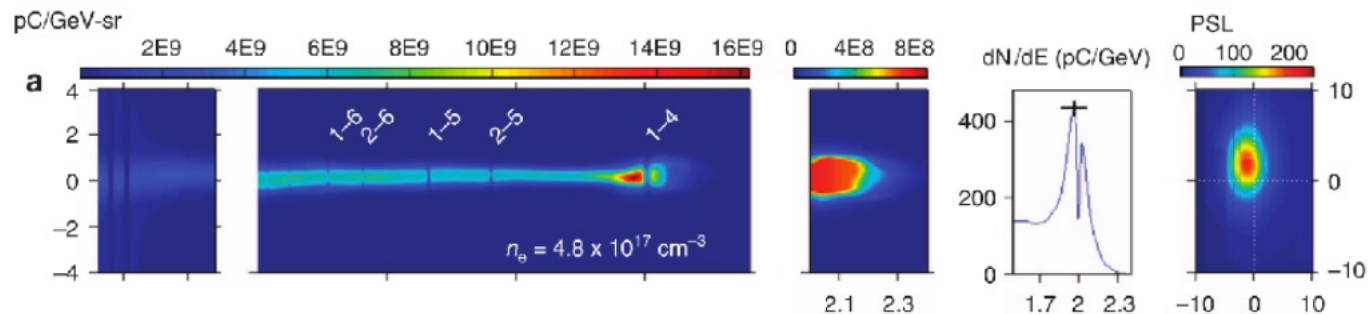
Received 2 Dec 2012 | Accepted 8 May 2013 | Published 11 Jun 2013

DOI: 10.1038/ncomms2988

OPEN

## Quasi-monoenergetic laser-plasma acceleration of electrons to 2 GeV

Xiaoming Wang<sup>1</sup>, Refal Zgadzaj<sup>1</sup>, Neil Fazel<sup>1</sup>, Zhengyan Li<sup>1</sup>, S. A. Yi<sup>1</sup>, Xi Zhang<sup>1</sup>, Watson Henderson<sup>1</sup>, Y.-Y. Chang<sup>1</sup>, R. Korzekwa<sup>1</sup>, H.-E. Tsai<sup>1</sup>, C.-H. Pai<sup>1</sup>, H. Quevedo<sup>1</sup>, G. Dyer<sup>1</sup>, E. Gaul<sup>1</sup>, M. Martinez<sup>1</sup>, A. C. Bernstein<sup>1</sup>, T. Borger<sup>1</sup>, M. Spinks<sup>1</sup>, M. Donovan<sup>1</sup>, V. Khudik<sup>1</sup>, G. Shvets<sup>1</sup>, T. Ditmire<sup>1</sup> & M. C. Downer<sup>1</sup>



X. Wang, Nature Communications, 4, 1988 (2013)

- 2 GeV
- $n_e = 4.8 \times 10^{17} \text{ cm}^{-3}$ , 7 cm gas cell
- $P = 1000 \text{ TW}$  “Texas PetaWatt” at University of Texas

*Courtesy of Stuart Mangles, CAS 2019*

Plasma acceleration



# Some experimental results

## Experiments at the energy frontier: 2014

Accepted Paper

### Multi-GeV electron beams from capillary-discharge-guided subpetawatt laser pulses in the self-trapping regime

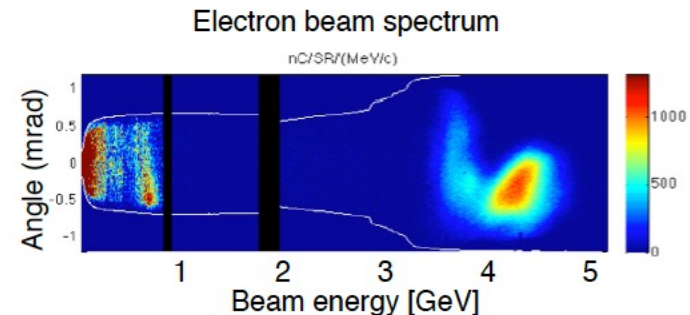
Phys. Rev. Lett.

W. P. Leemans, A. J. Gonsalves, H. S. Mao, K. Nakamura, C. Benedetti, C. B. Schroeder, Cs. Tóth, J. Daniels, D. E. Mittelberger, S. S. Bulanov, J.-L. Vay, C. G. R. Geddes, and E. Esarey

Accepted 21 October 2014

#### ABSTRACT

Multi-GeV electron beams with energy up to 4.2-GeV, 6-1% rms energy spread, 6 nC/picoCoulomb charge, and 0.3% (milliradian) rms divergence have been produced from a 8.5-cm-long capillary discharge waveguide with a plasma density of approx  $7 \times 10^{17} \text{ cm}^{-3}$ , powered by laser pulses with peak power up to 0.3-PW. Preformed plasma waveguides allow the use of lower laser power compared to unguided plasma structures to achieve the same electron beam energy. Detailed comparison between experiment and simulation indicates the sensitivity in this regime of the guiding and acceleration in the plasma structure to input intensity, density, and near-field laser mode profile.

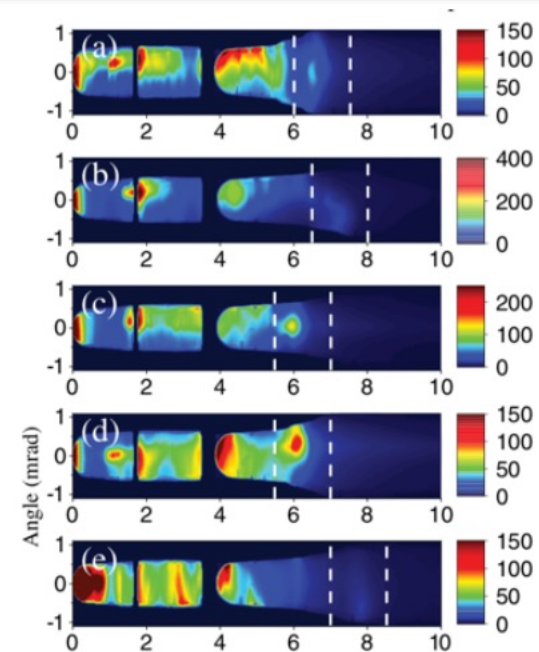
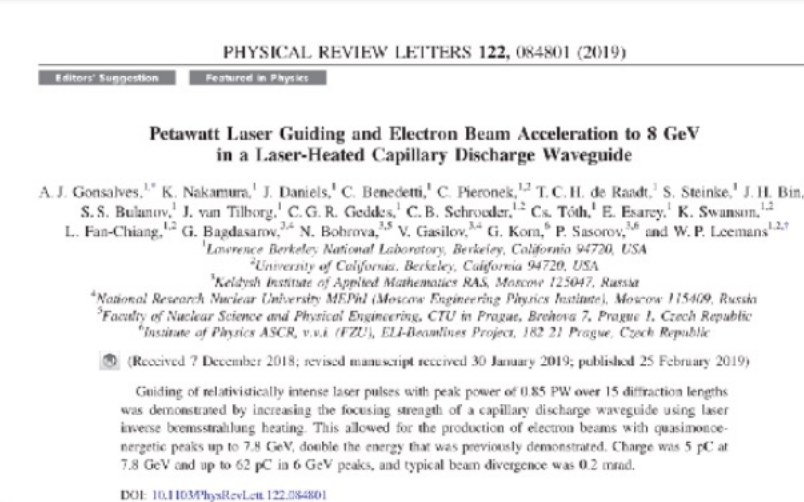


- 4 GeV
- $n_e = 7 \times 10^{17} \text{ cm}^{-3}$ , 9 cm capillary discharge waveguide
- $P = 300 \text{ TW}$  “Bella” at LBNL

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration

# Some experimental results

## Experiments at the energy frontier: 2019



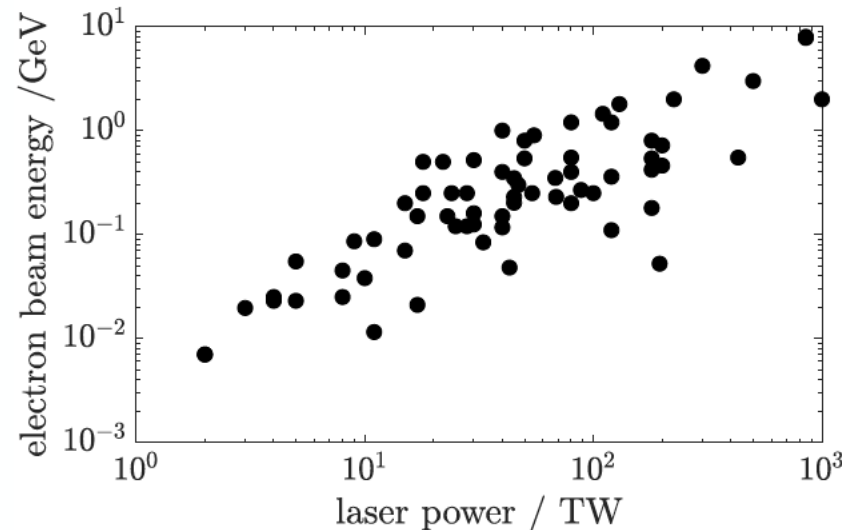
- 7.8 GeV
- $n_e = 7 \times 10^{17} \text{ cm}^{-3}$ , 9 cm capillary discharge waveguide
- $P = 850 \text{ TW}$  “Bella” at LBNL

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration



# Some experimental results

But science isn't about collecting World Records.... Can we extract some physics from the data trends?

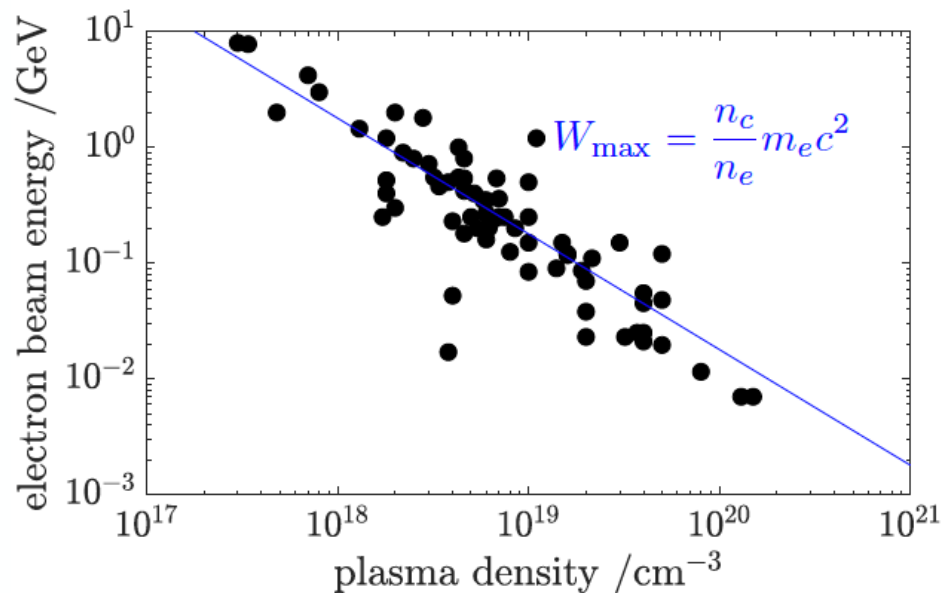


- Collection of data from a variety of experiments
  - (not just the record breakers, but probably the highest beam each experiment was capable of producing)
    - Trend is: higher laser power = higher electron energy
    - What is physics behind this?

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration

# Some experimental results

Electron energy is limited by dephasing  
– move to lower densities



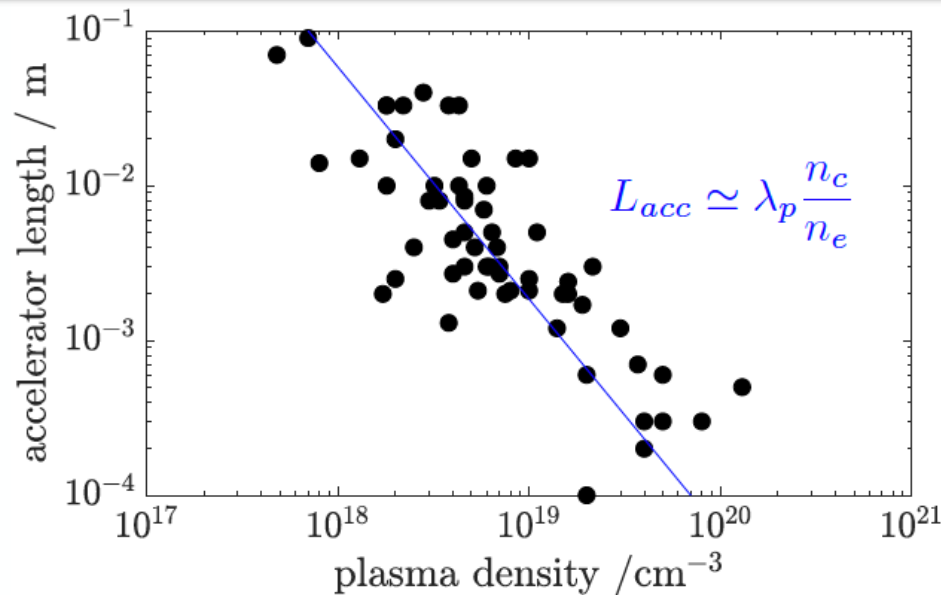
- Beam energy,  $W_{\text{max}}$ , is inversely proportional to plasma density as expected for dephasing

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration

# Some experimental results

Electron energy is limited by dephasing

– move to lower densities and longer accelerators

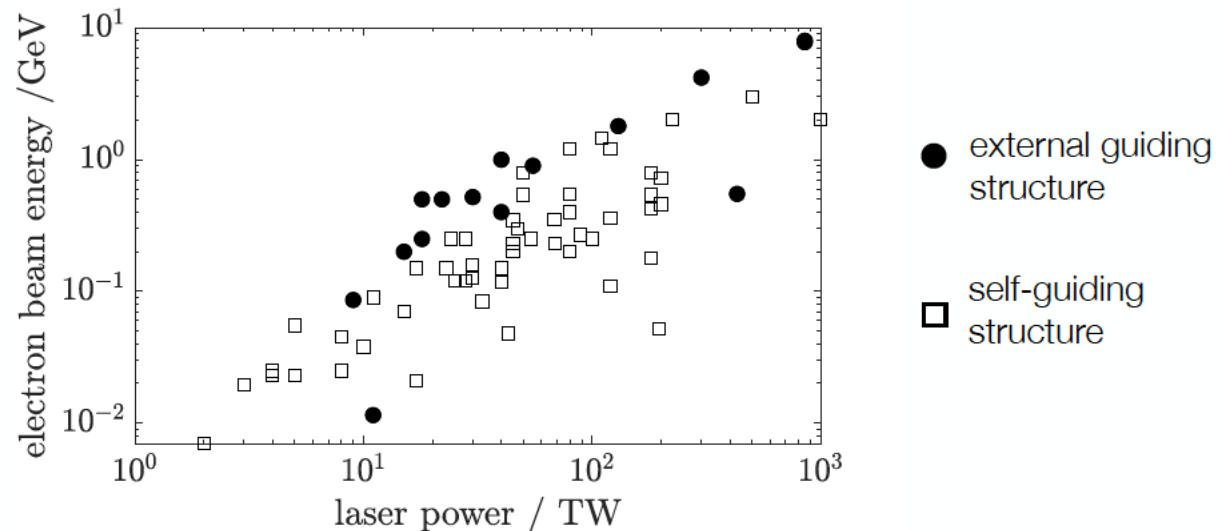


- Accelerator length increases for lower density experiments
  - data lies close to dephasing length (even for simplest linear regime expression)

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration

# Some experimental results

To guide or not to guide?

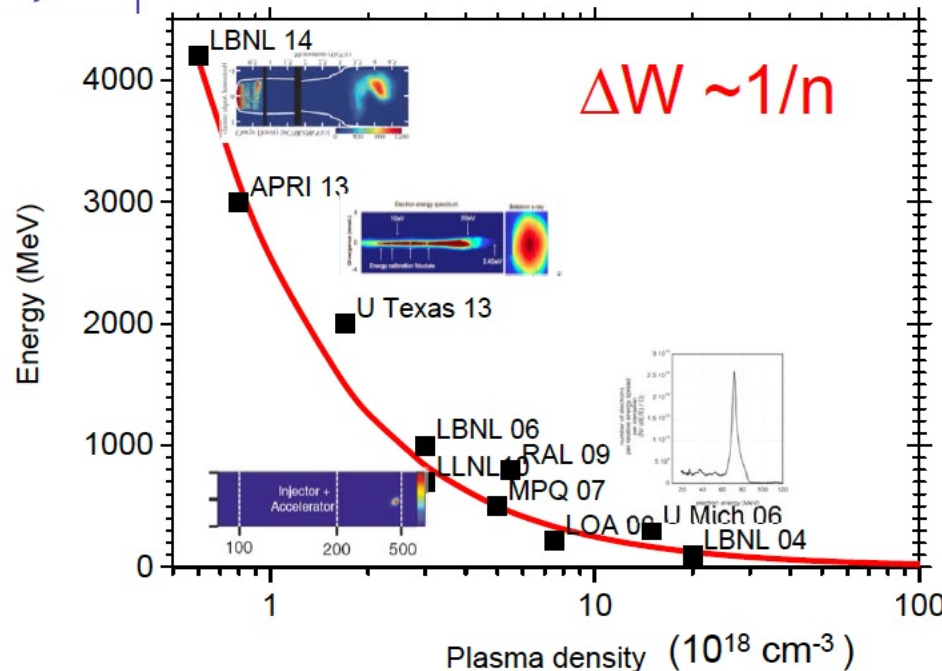


- Data shows that experiments in pre-formed plasma structures are “best” performers
  - i.e. for a given laser power the highest energy beams produced come from guided experiments
  - one (common) explanation is that guiding structure is less lossy

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration

# Dephasing length

Energy gain is large at low plasma density over a long distance



Non Linear regime with injection of plasma electrons

Energies above GeV reached for PW laser power: UTexas13, APRI13: 2 gaz jets

LBNL14 also includes channel guiding

- ➡ Energy increases for lower plasma density
- ➡ At low enough density, self-injection stops, additional laser power or external injection should be used

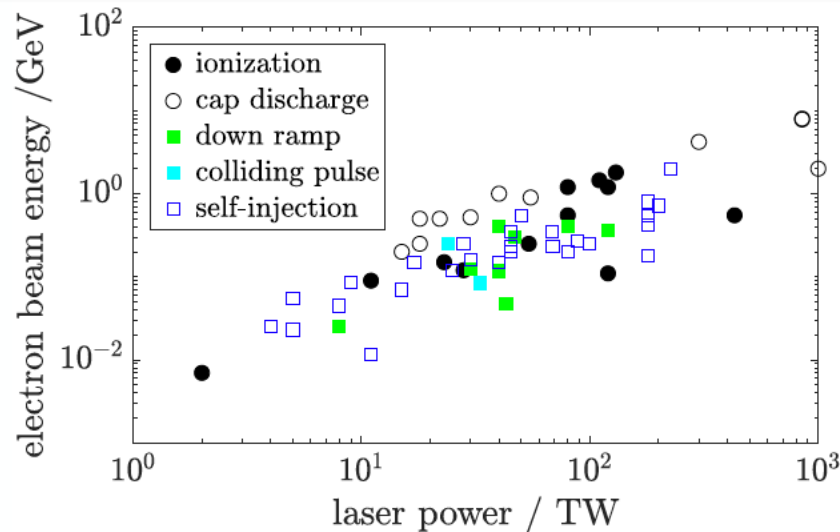
15



Courtesy of Brigitte Cros, CAS 2019  
Plasma acceleration

# Some experimental results

To inject or not to inject?



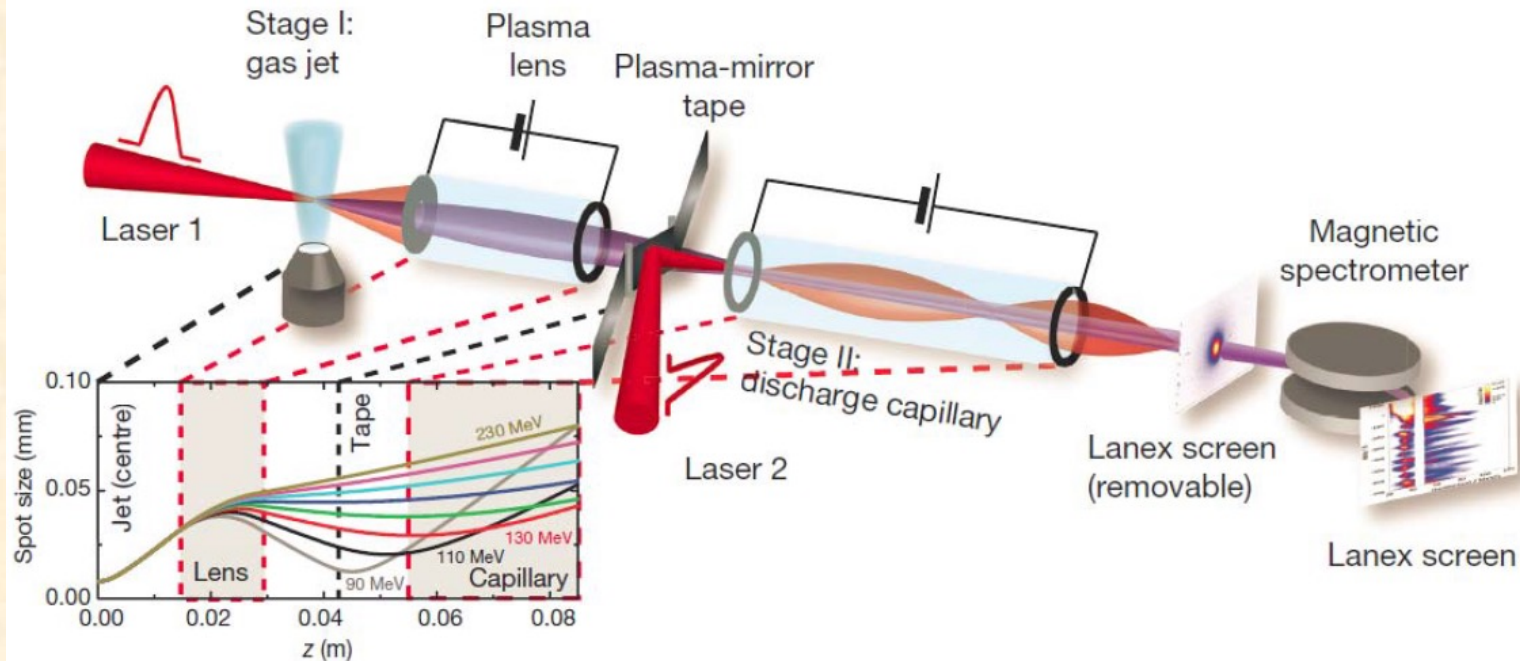
- some ionisation injection experiments also lie at upper edge of distribution
  - data too noisy for a definitive answer, but an interesting research question

*Courtesy of Stuart Mangles, CAS 2019*  
Plasma acceleration



# Multi-stage

## Coupling an electron source to a plasma accelerator



S. Steinke et al., Nature 2016

B. Cros, CAS HGWA Sesimbra, March 2019



Courtesy of Brigitte Cros, CAS 2019  
Plasma acceleration

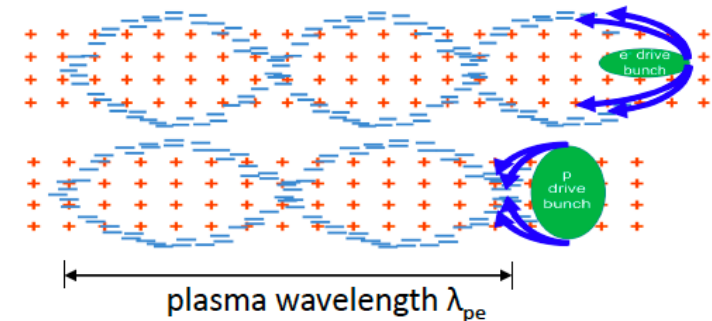
# **BEAM DRIVEN PLASMA ACCELERATION**

# Beam driven plasma acceleration

## Plasma Wakefield Acceleration

Different ways to excite the wakes - most commonly used:

- Laser bunches, Electron beams, Protons bunches



A plasma of density  $n_{pe}$  is characterized by the plasma frequency

$$\omega_{pe} = \sqrt{\frac{n_{pe} e^2}{m_e \epsilon_0}} \Rightarrow \frac{c}{\omega_{pe}} \dots \text{unit of plasma [m]} \quad k_{pe} = \frac{\omega_{pe}}{c}$$

$$\text{Example: } n_{pe} = 7 \times 10^{14} \text{ cm}^{-3} \text{ (AWAKE)} \Rightarrow \omega_{pe} = 1.25 \times 10^{12} \text{ rad/s} \Rightarrow \frac{c}{\omega_{pe}} = 0.2 \text{ mm} \Rightarrow k_{pe} = 5 \text{ mm}^{-1}$$

This translates into a wavelength of the plasma oscillation

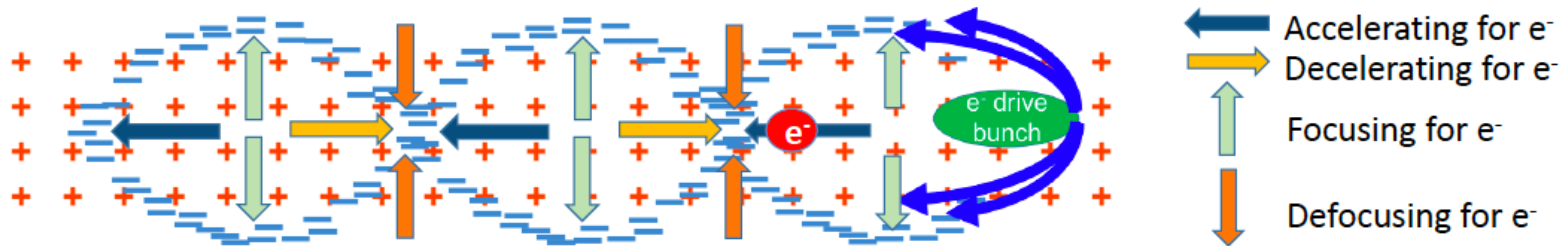
$$\lambda_{pe} = 2\pi \frac{c}{\omega_{pe}} \Rightarrow \lambda_{pe} \approx 1 \text{ mm} \sqrt{\frac{10^{15} \text{ cm}^{-3}}{n_{pe}}}$$

$$\lambda_{pe} = 1.2 \text{ mm} \Rightarrow \text{Cavities with mm size!}$$

7

# Beam driven plasma acceleration

## Wakefields



How strong can the fields be?

- The plasma oscillation leads to a longitudinal accelerating field. The maximum accelerating field (wave-breaking field) is:
- The ion channel left on-axis, where the beam passes, induces an **ultra-strong focusing field**:

$$e E_{WB} = 96 \frac{V}{m} \sqrt{\frac{n_{pe}}{cm^{-3}}}$$

$$g = 960 \pi \frac{n_{pe}}{10^{14} cm^{-3}} \frac{T}{m}$$

**Example:**  $n_{pe} = 7 \times 10^{14} cm^{-3}$  (AWAKE)  $\rightarrow eE_{WB} = 2.5$  GV/m  $\rightarrow g = 21$  kT/m  
**Example:**  $n_{pe} = 7 \times 10^{17} cm^{-3}$   $\rightarrow eE_{WB} = 80$  GV/m  $\rightarrow g = 21$  MT/m

# Beam driven plasma acceleration

## Record Acceleration, at SLAC: 42 GeV

Final Focus Test Beam Facility, **FFTB** at SLAC

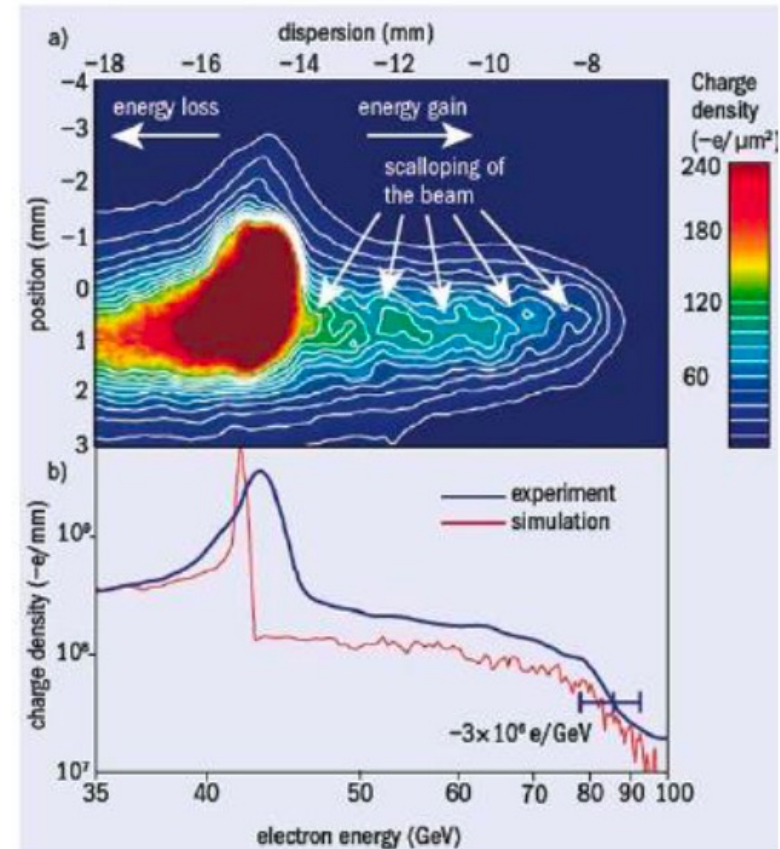
I. Blumenfeld et al, Nature 455, p 741 (2007)

Gaussian electron beam with 42 GeV, 3nC @ 10 Hz,  $\sigma_x = 10\mu\text{m}$ , 50 fs

85cm Lithium vapour source,  $2.7 \times 10^{17} \text{cm}^{-3}$

→ Accelerated electrons from 42 GeV to 85 GeV in 85 cm.

→ Reached accelerating gradient of **52 GeV/m**



*Courtesy of Edda Gschwendtner, CAS 2019*  
Plasma acceleration



# Beam driven plasma acceleration

## SLAC – FACET

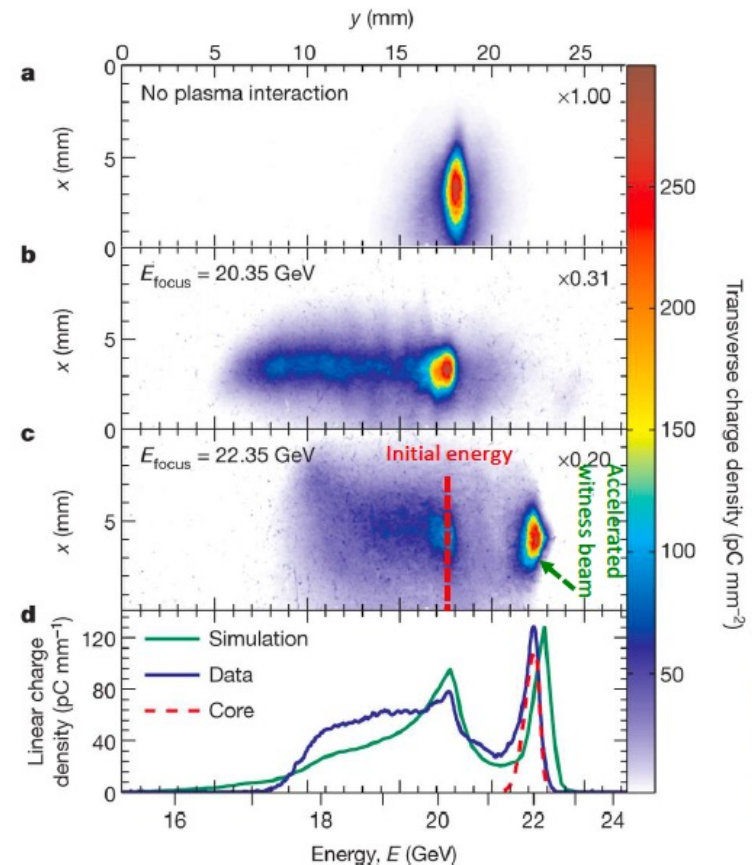
### High-Efficiency acceleration of an electron beam in a plasmas wakefield accelerator, 2014

M. Litos et al., doi, Nature, 6 Nov 2014, 10.1038/nature 13882

- Laser ionized Lithium vapour plasma cell:
  - 36 cm long, Density:  $5 \times 10^{16} \text{ cm}^{-3}$ ,  $\lambda_{\pi} = 200 \text{ }\mu\text{m}$
- Drive and witness beam:
  - 20.35 GeV, D and W separated by  $160 \text{ }\mu\text{m}$
  - 1.02nC (D), 0.78nC (W)

First demonstration of a high-efficiency, low energy-spread plasma wakefield acceleration experiment:

- 70 pC of charge accelerated
- 2 GeV energy gain
- 5 GeV/m gradient
- **Up to 30% transfer efficiency**
- **~2% energy spread**



23

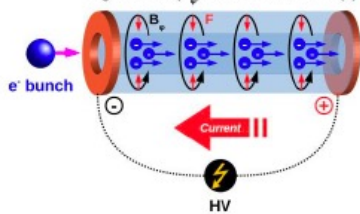
Courtesy of Edda Gschwendtner, CAS 2019  
Plasma acceleration

# Plasma lens

## SPARCLAB, Plasma Lens Experiment

### Plasma Lens

Magnetic Field ( $B_z$ ) vs Force on electrons ( $F$ )



Beam focusing by azimuthal magnetic field generated by the discharge current density

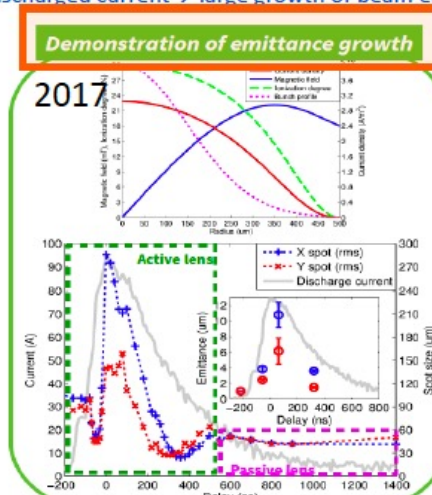
$$B_\phi(r) = \frac{\mu_0}{r} \int_0^r J(r') r' dr'$$

### Experiment:

127MeV, 50pC,  $\sigma_t=1.3\text{ps}$ ,  $\epsilon_N \sim 1\text{ mm mrad}$ ,  
 $\sigma_x = 110\mu\text{m}$ .

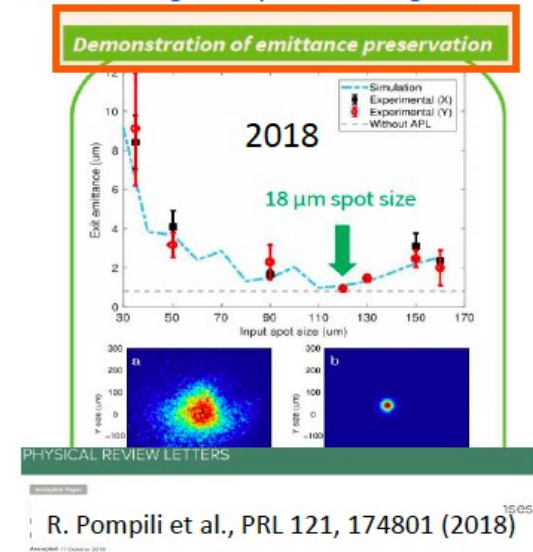
Capillary discharge plasma cell, 3cm,  
 $R_0=500\mu\text{m}$ ,  $I=100\text{A}$ ,  $V=20\text{kV}$ ,  $\text{H}_2$  gas,  $n_e = 9 \times 10^{16} \text{cm}^{-3}$ ,

→ Focusing is non-linear due to non-uniformity of the discharged current → large growth of beam emittance

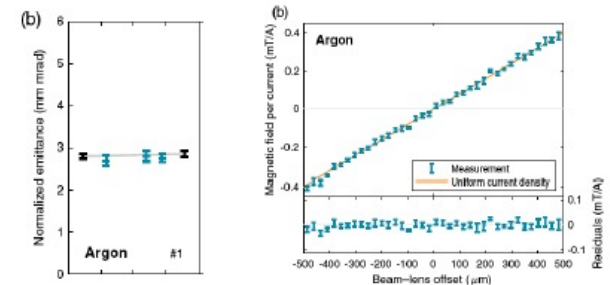


R. Pompili et al., Applied Physics Letters 110.10 (2017):104101  
 A. Marocchino et al., Applied Physics Letters 111.18(2017):184101

→ Change plasma discharge  
 → Enhancing linearity of the focusing field.



C. Lindstroem et al., Emittance Preservation in Aberration-Free Active Plasma Lens, PRL 121, 194801 (2018)



31

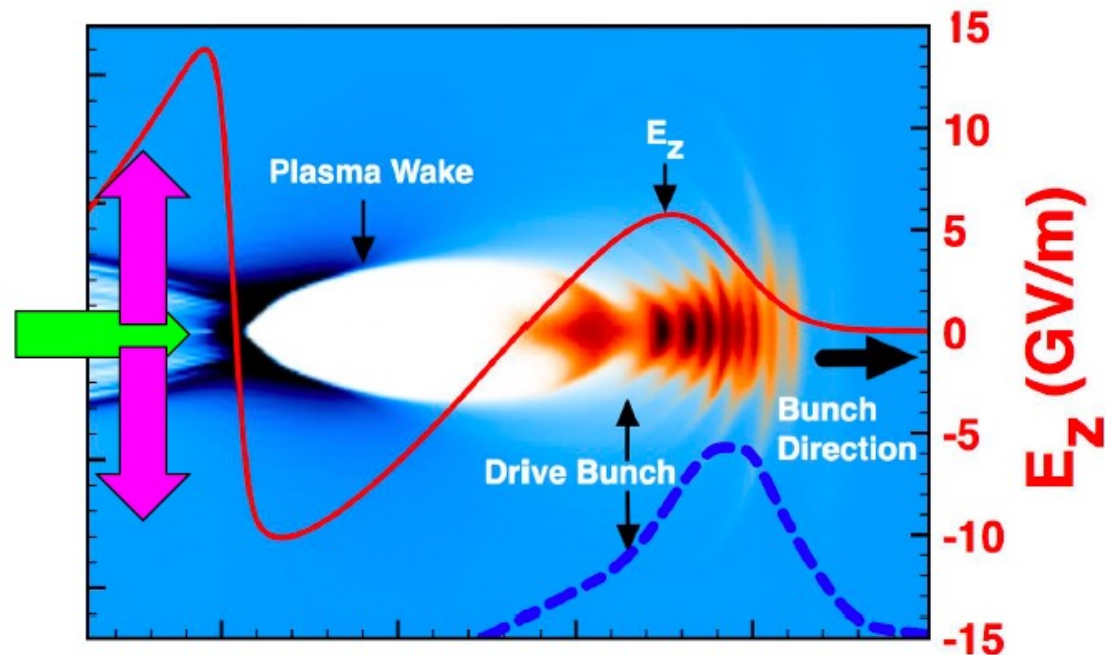
# POSITRON ACCELERATION

# Positron acceleration

## Positron Acceleration

- Interested in using positrons for high energy linear colliders:
  - Parameters for positrons: **high energy, high charge, low emittance.**

Electron-driven blowout wakes:



But the field is **defocusing** in this region.

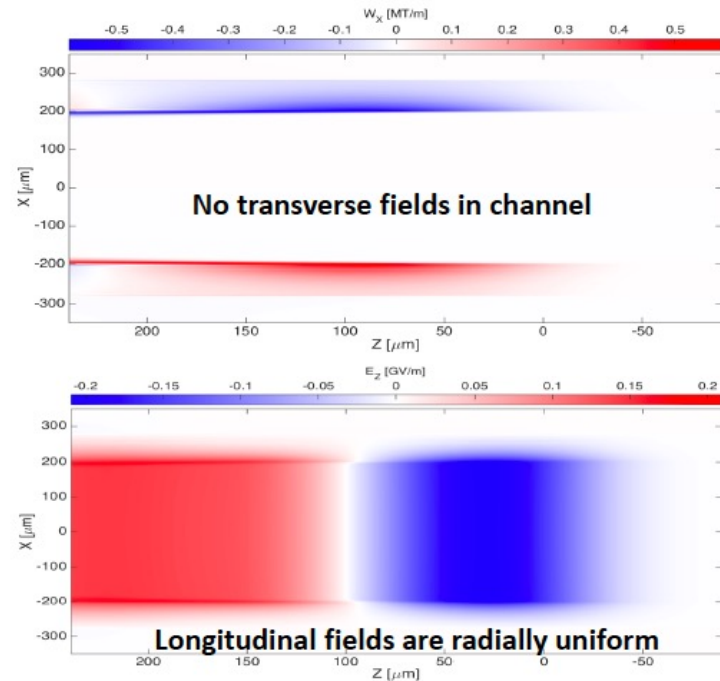
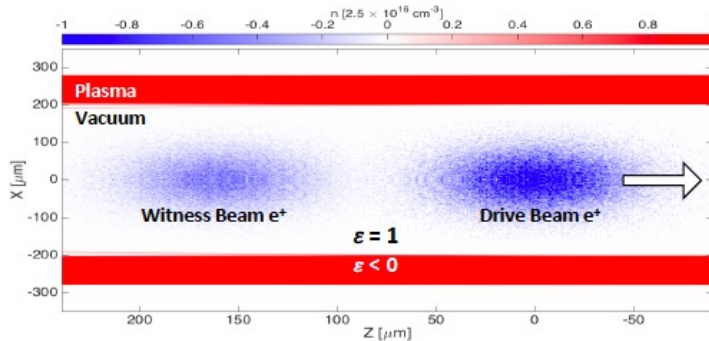
*Courtesy of Edda Gschwendtner, CAS 2019*  
Plasma acceleration



# Positron acceleration

## Positron Acceleration in Hollow Channel at FACET

- There is no plasma on-axis, and therefore no complicated forces from plasma electrons streaming through the beam.
- Treat the plasma as dielectric



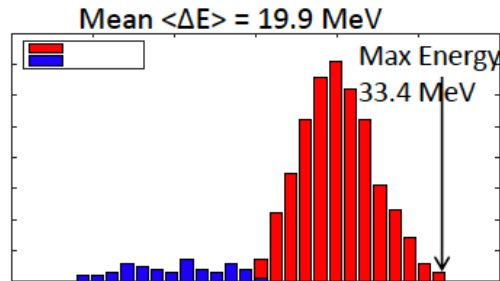
*Courtesy of Edda Gschwendtner, CAS 2019*  
Plasma acceleration



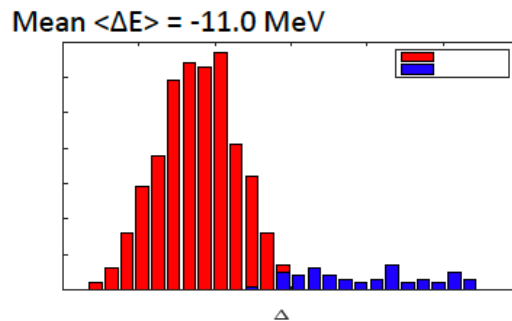
# Positron acceleration

## Positron Acceleration in Hollow Channel at FACET, 2016, 2018

### First Demonstration of Acceleration in Hollow channel



Witness beam gains energy from the wake.



Drive beam transfers energy to witness beam.

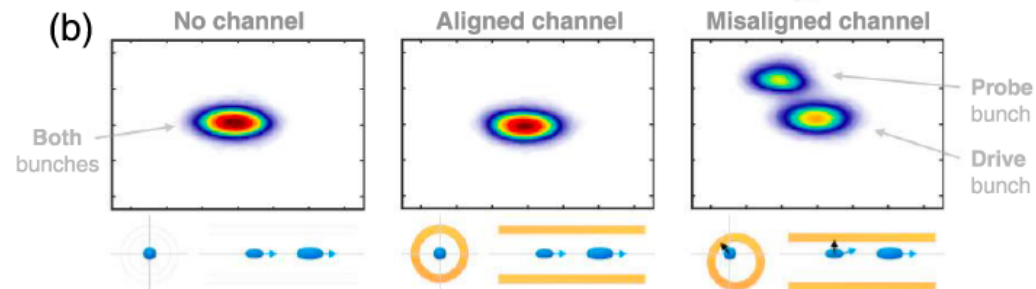
S. Gessner et. al. *Nat. Comm.* 7, 11785 (2016)

### Measurement of transverse wakefields in hollow channel

→ the result agrees with theoretical calculation:

$$10^6 \text{ V}/(\text{pC m mm})$$

Or about 10,000 times stronger than the wakefields in CLIC!



C. A. Lindstrøm et. al. *Phys. Rev. Lett.* 120 124802 (2018).

# **PROTON DRIVEN PLASMA ACCELERATION**

# Protons as a driver

# Energy Budget for High Energy Plasma Wakefield Accelerators

### Drive beams:

Lasers:  $\sim 40$  J/pulse

Electron drive beam: 30 J/bunch

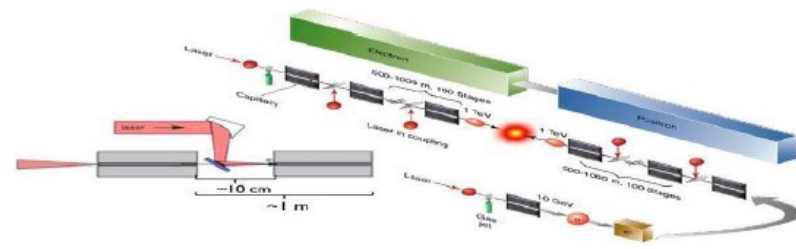
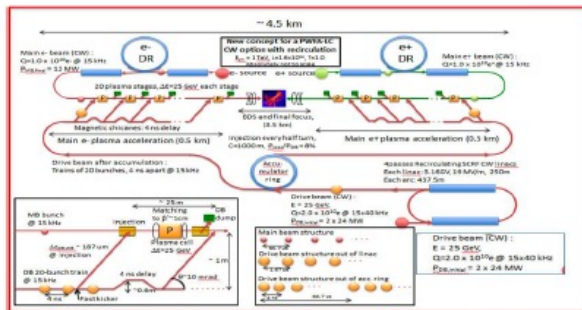
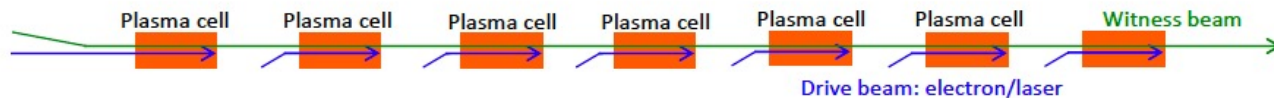
Proton drive beam: SPS 19kJ/pulse, LHC 300kJ/bunch

**Witness beams:**

Electrons:  $10^{10}$  particles @ 1 TeV ~few kJ

**To reach TeV scale:**

- **Electron/laser driven PWA:** need several stages, and challenging wrt to relative timing, tolerances, matching, etc...
  - effective gradient reduced because of long sections between accelerating elements....

E. Adli *et. al.*, arXiv:1308.1145 [physics.acc-ph]C. B. Schroeder *et. al.* Phys. Rev. ST Accel. Beams **13**, 101301

43

Courtesy of Edda Gschwendtner, CAS 2019  
Plasma acceleration

# Protons as a driver

## Seeded Self-Modulation of the Proton Beam

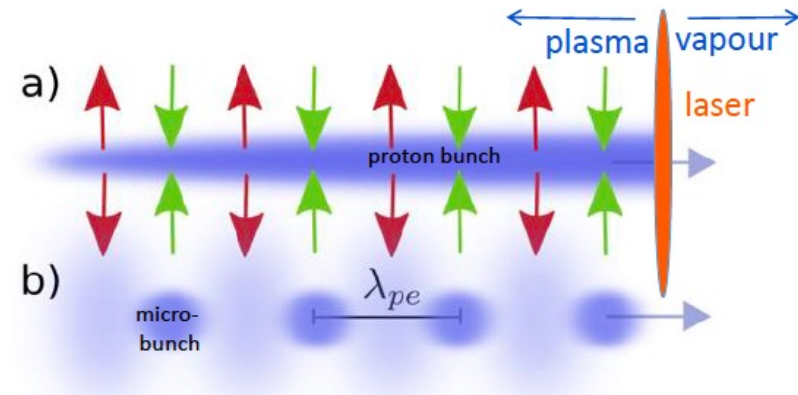
In order to create plasma wakefields efficiently, the drive bunch length has to be in the order of the plasma wavelength.

CERN SPS proton bunch: very long! ( $\sigma_z = 12$  cm)  $\rightarrow$  much longer than plasma wavelength ( $\lambda = 1$  mm)

N. Kumar, A. Pukhov, K. Lotov,  
PRL 104, 255003 (2010)

### Self-Modulation:

- a) Bunch drives wakefields at the initial seed value when entering plasma.
  - Initial wakefields act back on the proton bunch itself.  $\rightarrow$  On-axis dens is modulated.  $\rightarrow$  Contribution to the wakefields is  $\propto n_b$ .
- b) Density modulation on-axis  $\rightarrow$  micro-bunches.
  - Micro-bunches separated by plasma wavelength  $\lambda_{pe}$ .
  - drive wakefields resonantly.



### $\rightarrow$ Seeded Self-Modulation

#### AWAKE: Seeding of the instability by

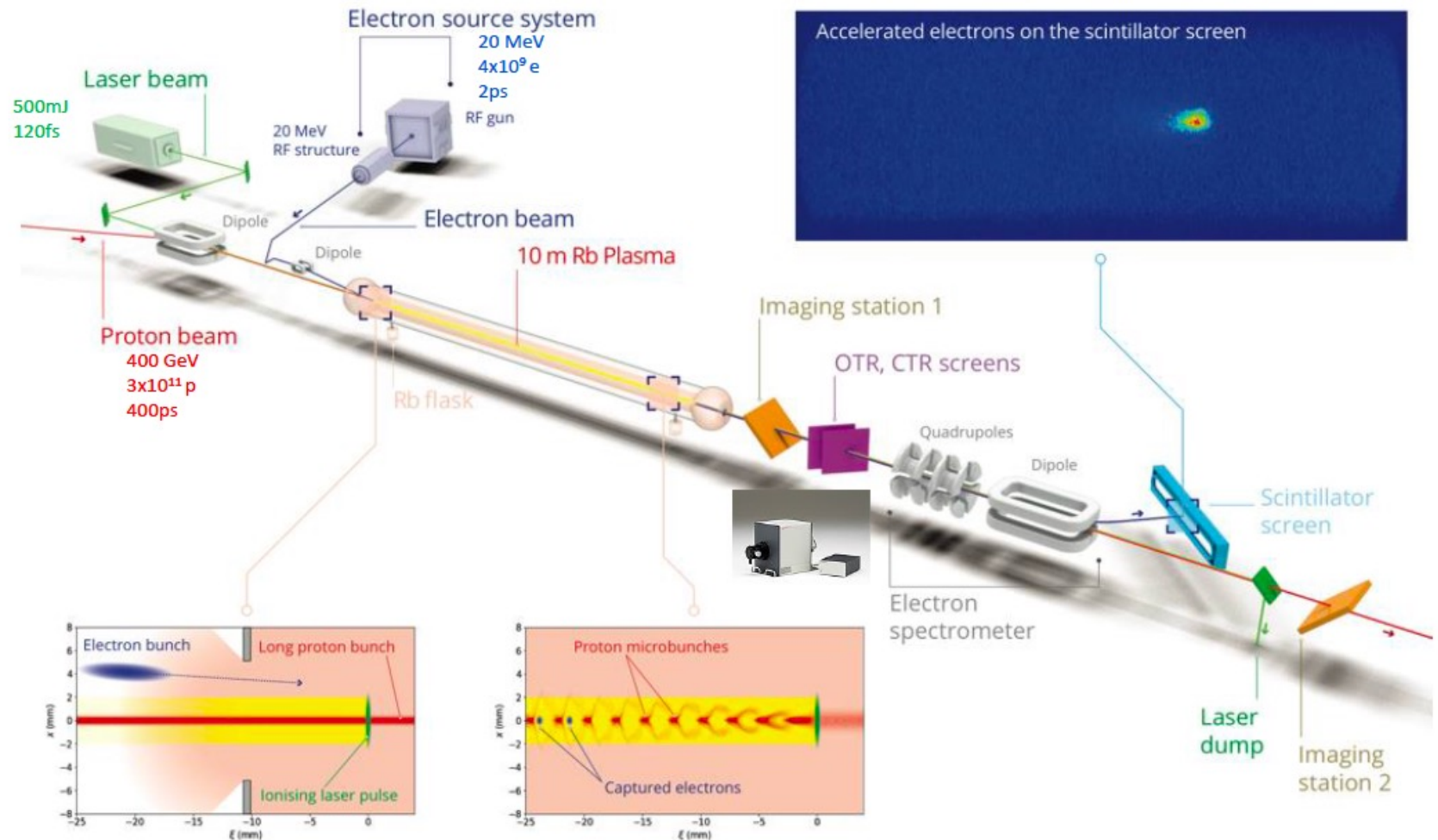
- Placing a **laser** close to the center of the proton bunch
- Laser ionizes vapour to produce plasma
- Sharp start of beam/plasma interaction
- $\rightarrow$  Seeding with ionization front

45



# Protons as a driver

## AWAKE Experiment



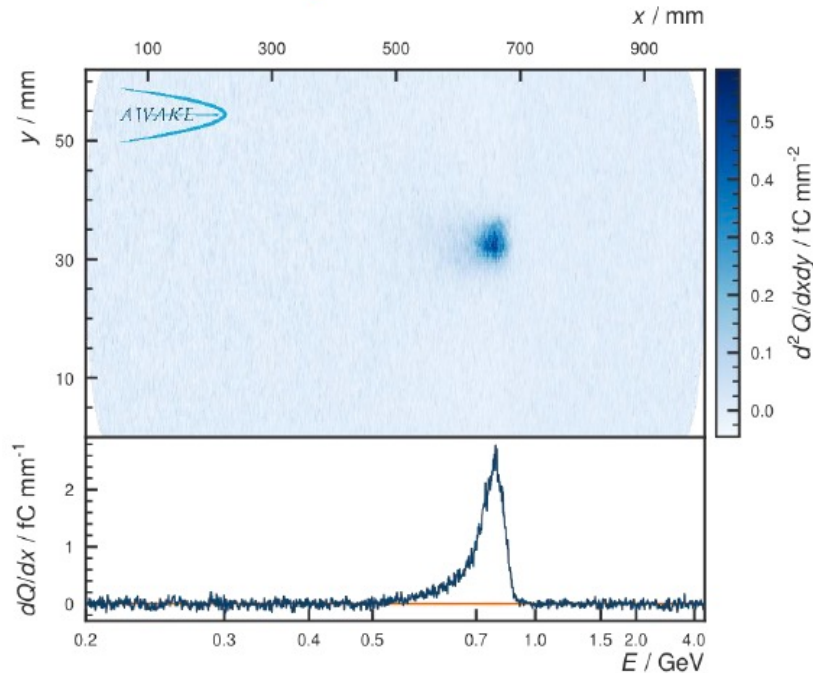
Courtesy of Edda Gschwendtner, CAS 2019  
Plasma acceleration



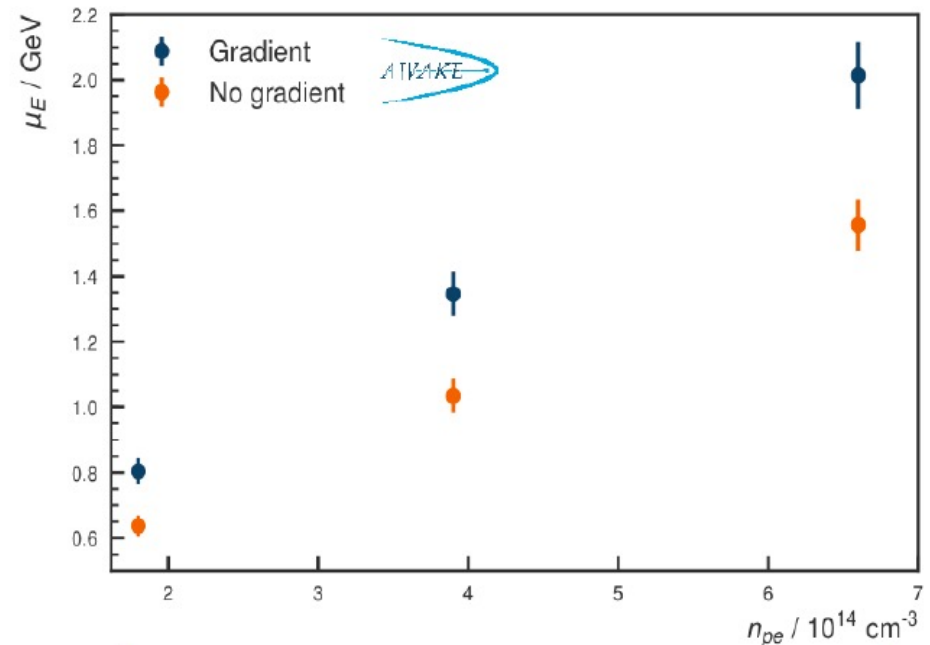
# Protons as a driver

## Electron Acceleration Results, 2018

Results from May 2018 Run



Event at  $n_{pe} = 1.8 \times 10^{14} \text{ cm}^{-3}$  with 5%/10m density gradient.



→ Acceleration up to 2 GeV has been achieved.

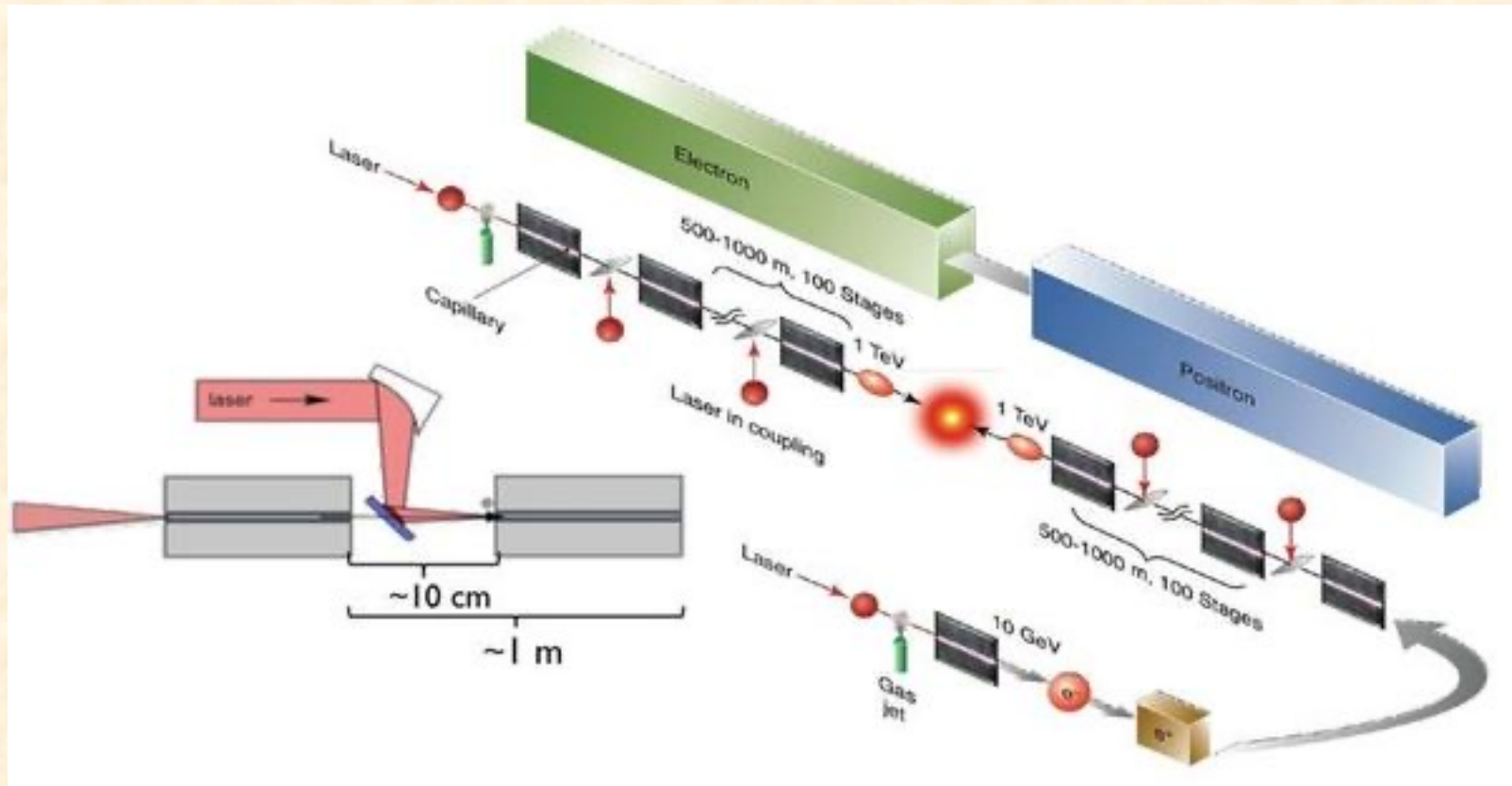
AWAKE Collaboration, Nature, doi:10.1038/s41586-018-0485-4 (2018)

54

Courtesy of Edda Gschwendtner, CAS 2019  
Plasma acceleration

# **A PLASMA COLLIDER?**

# A plasma collider proposal

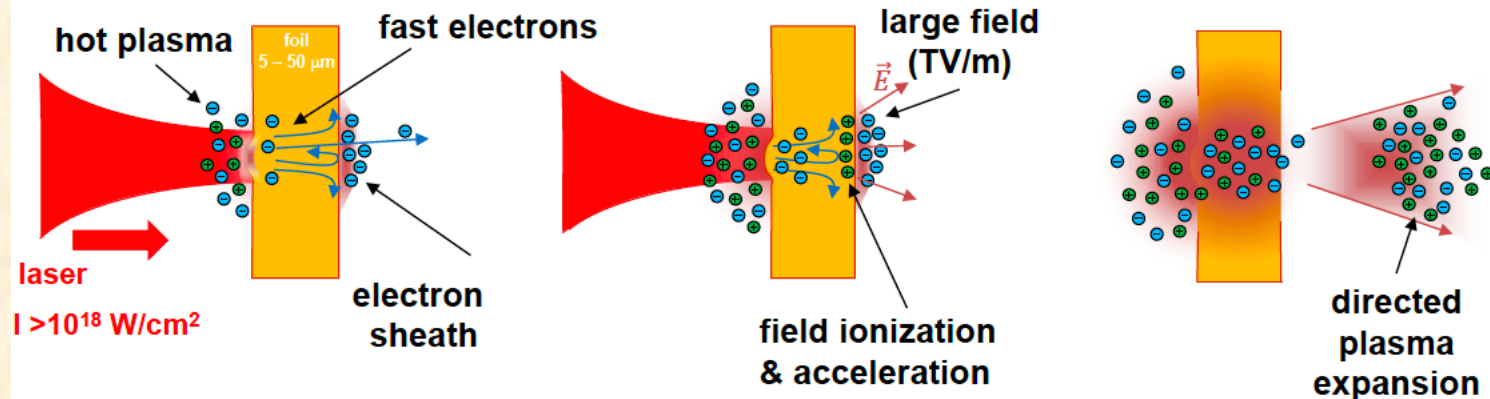


- A concept of plasma collider has been proposed.  
<https://physicstoday.scitation.org/doi/10.1063/1.3099645>

# **ACCELERATION OF IONS**

# Acceleration of ions

TNSA is the most widely used and robust acceleration scheme



- **intense:**  $10^{10} - 10^{13}$  protons
  - initial bunch duration  $\leq 1 \text{ ps}$
  - source size  $< 100 \mu\text{m}$
- **ultra-low emittance\***
  - $< 0.01 \text{ mm mrad trans.}$
  - $< 10^{-4} \text{ eV s long.}$
- **compact:**  $\text{MV}/\mu\text{m}$

- **divergence:**  $\leq 30 \text{ deg}$  (half angle)
- **continuous exp. spectrum**
- **disturbed environment**
  - electrons
  - large background:  $\gamma$ , X-rays, EMP

\*T. E. Cowan *et al.*, PRL **92**, 204801 (2004)

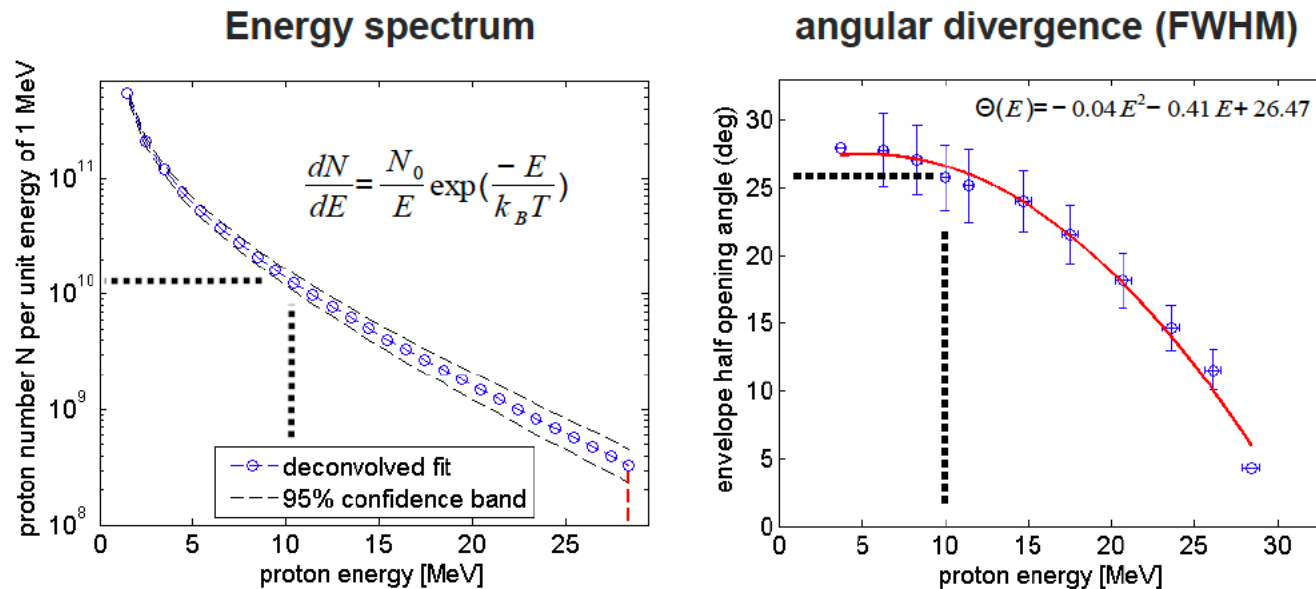
GSI Helmholtzzentrum für Schwerionenforschung GmbH

Courtesy of Vincent Bagnoud, CAS 2019  
Plasma acceleration



# Acceleration of ions

Typical properties of TNSA beams exhibit a broad spectrum and large angular divergence



- ♦ detection of full proton beam via RIS\* @4cm behind source
- ♦ source size @10MeV: approx. 50 $\mu$ m

\*F. Nürnberg *et al.*, RSI **80**, 033301

GSI Helmholtzzentrum für Schwerionenforschung GmbH

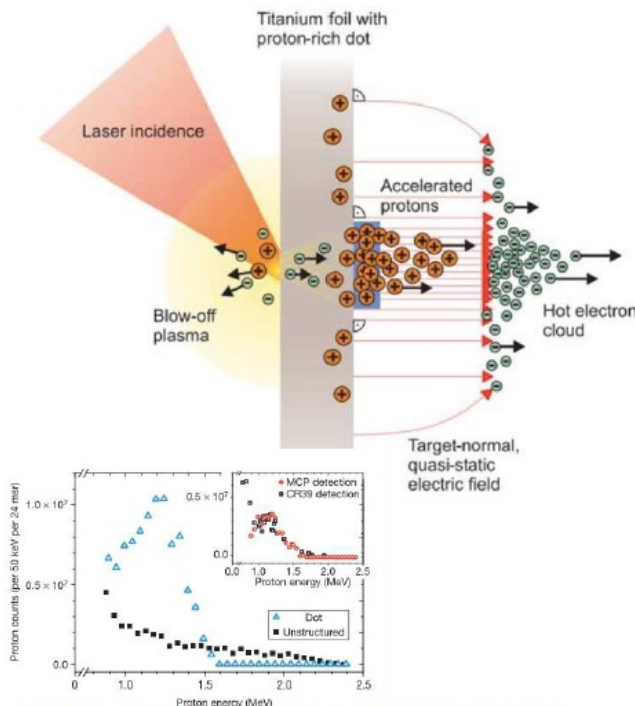
5

Courtesy of Vincent Bagnoud, CAS 2019  
Plasma acceleration

# Acceleration of ions

Ion acceleration mechanisms:  
Small energy spread using TNSA?

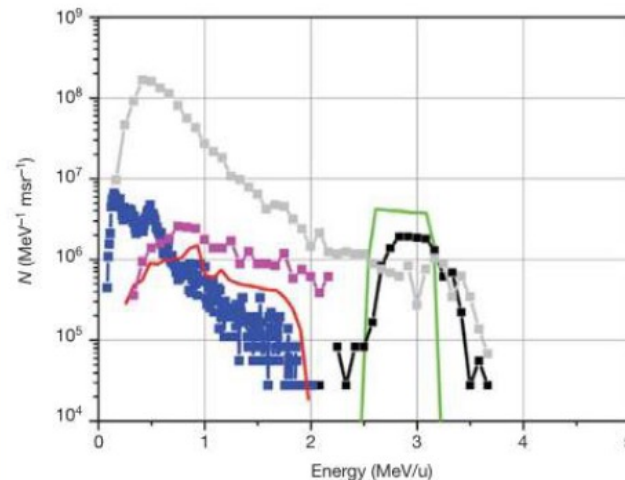
## Structured targets (CH microdot)



H Schworer, et al, Nature, 439, 445 (2006);  
APL Robinson and P Gibbon, PRE, 75, 015401 (2007)

## Complex target preparation:

“an ultrathin layer of graphitic carbon, formed from catalytic decomposition of adsorbed hydrocarbon impurities on a 20 mm palladium foil.”



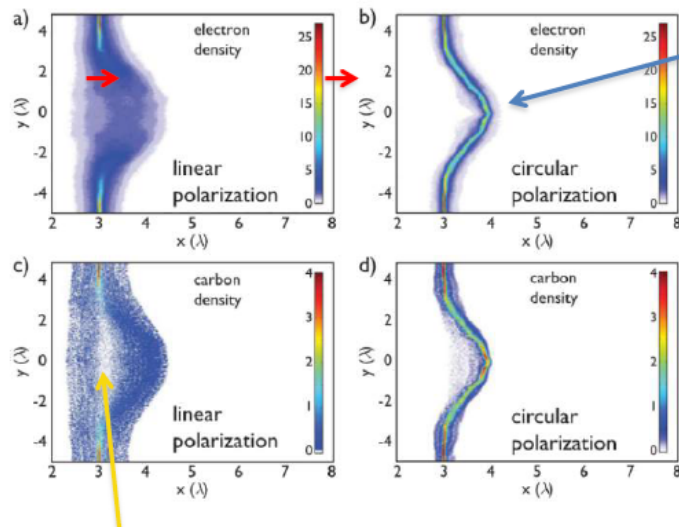
BM Hegelich, et al, Nature, 439, 441 (2006)

*Courtesy of Louise Willingale, CAS 2019*  
Plasma acceleration

# Acceleration of ions

## Advanced ion acceleration mechanisms: Radiation Pressure Acceleration (RPA)

A Henig, et al, PRL, 103, 245003 (2009)



Linear polarization heats electrons strongly and explodes foil, preventing the “light-sail” from forming – TNSA instead.

Esirkepov, et al, PRL, 92, 175003 (2004)

Laser light pressure pushes entire electron volume of a very thin foil forward forming the acceleration field:

“Light Sail” regime

The ions follow the electrons – all experience same field  $\rightarrow$  same final energy.

- ✓ Excellent ion energy scaling with laser intensity
- ✓ Excellent energy conversion efficiency
- ✓ Quasi-monoenergetic acceleration

- ✗ Very thin targets difficult to handle
- ✗ Requires challenging laser parameters:
  - Very small laser pre-pulse
  - Circular polarization
  - Large focal spot, increases the laser energy required
- ✗ Experimental demonstrations have been so far disappointing

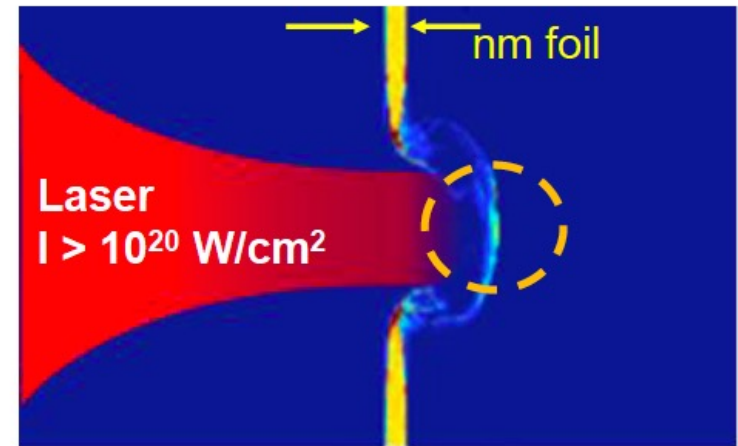
Courtesy of Louise Willingale, CAS 2019  
Plasma acceleration

# Acceleration of ions

RPA and BOA/RITA require ultrathin targets



- “advanced schemes” rely on a direct acceleration
- Very hard experimental conditions are necessary
  - thin foils are necessary (typically  $< 10$ 's nm)
  - electrons should remain cold – circularly polarized light is necessary
  - ultra-clean temporal profile of the laser pulse
- performance of simulations never confirmed experimentally



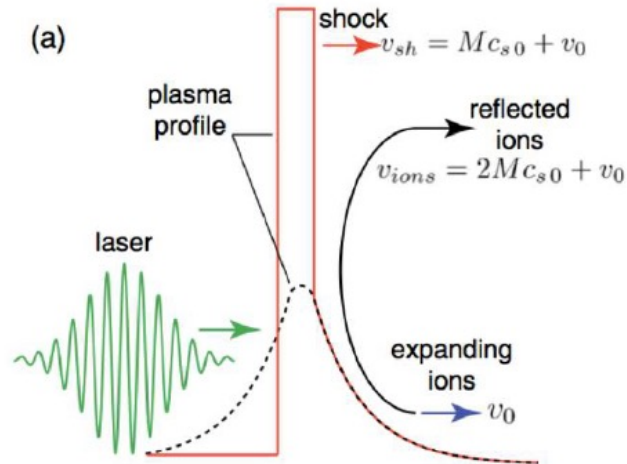
\* adapted from Robinson

*Courtesy of Vincent Bagnoud, CAS 2019*  
Plasma acceleration

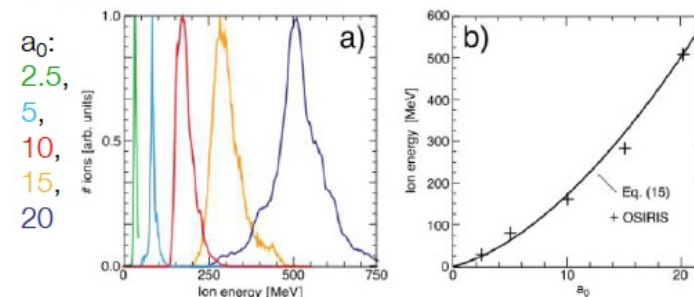


# Acceleration of ions

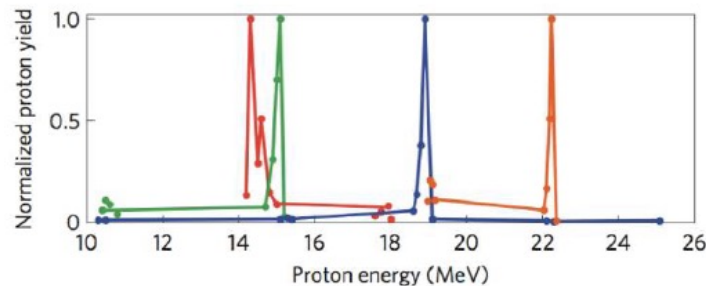
## Advanced ion acceleration mechanisms: Shock acceleration



Very promising theoretical energy scaling with  $a_0$ :



Demonstration of quasi-monoenergetic proton spectra using CO<sub>2</sub> ( $\lambda = 10 \mu\text{m}$ ) lasers:



Laser	$\lambda$	$n_c$	$a_0$
CO <sub>2</sub>	10 $\mu\text{m}$	$10^{19} \text{ cm}^{-3}$	2
Glass	1.053 $\mu\text{m}$	$10^{21} \text{ cm}^{-3}$	20
Ti:Sapph	800 nm	$1.1 \times 10^{21} \text{ cm}^{-3}$	50

D Haberberger, et al, Nature Physics, 8, 95 (2012);  
F Fiuza, et al, PRL, 109, 215001 (2012);  
F Fiuza, et al, Phys Plas, 20, 056304 (2013).

Courtesy of Louise Willingale, CAS 2019  
Plasma acceleration

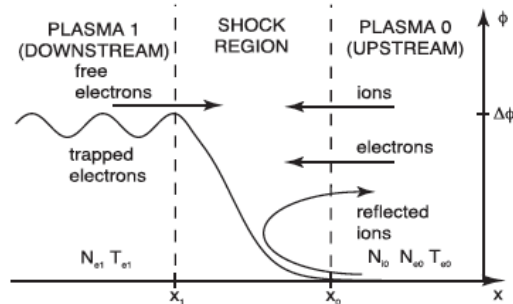


# Acceleration of ions

## Advanced ion acceleration mechanisms: Shock acceleration

Shock formation requires a high plasma electron temperature,  $T_{e1}$ .

F Fiuza, et al, Phys Plas, 20, 056304 (2013)



$$M_{cr} = \sqrt{2 \frac{T_{e1}}{T_{e0}} \left( \frac{1 + \mu_{e0}}{\frac{N_{e1}}{N_{e0}} \left( 1 - \mu_{e0} \frac{T_{e0}}{T_{e1}} \right)} + 1 \right)}$$

$$\mu_{e0} = \frac{m_e c^2}{k_B T_{e0}}$$

This requires strong laser absorption and places restrictions on the target size and scalelengths for optimum acceleration.

- ✓ Excellent ion energy scaling with laser intensity
- ✓ Quasi-monoenergetic acceleration
- ✓ Gas-jet useful for high-rep rate & low debris
- ✓ Experimentally demonstrated
- ✗ Requires challenging target parameters:
  - Very-high density gas jet / prepared target
  - Relativistic Transparency increases density requirement – even more difficult
  - Carefully designed density profile needed
- ✗ Large focal spot needed, increases the laser energy required
- ✗ Instabilities not studied

Courtesy of Louise Willingale, CAS 2019

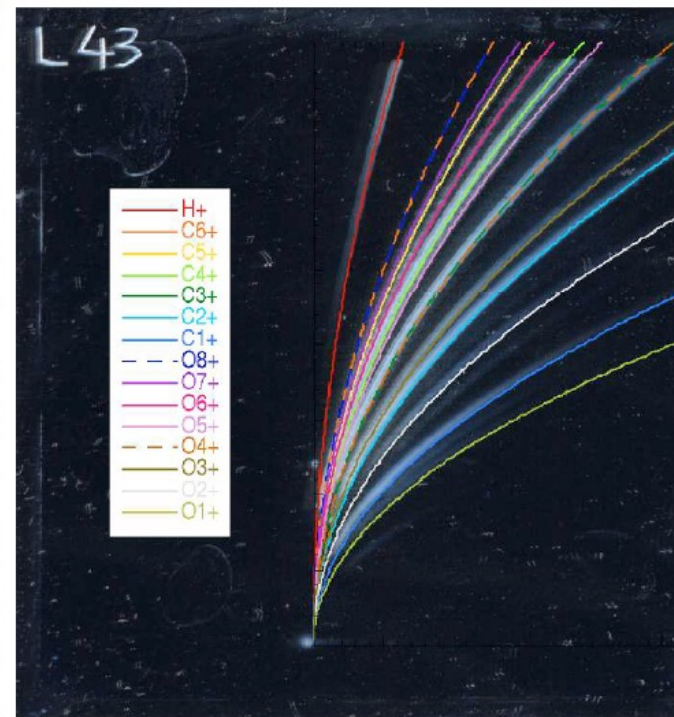
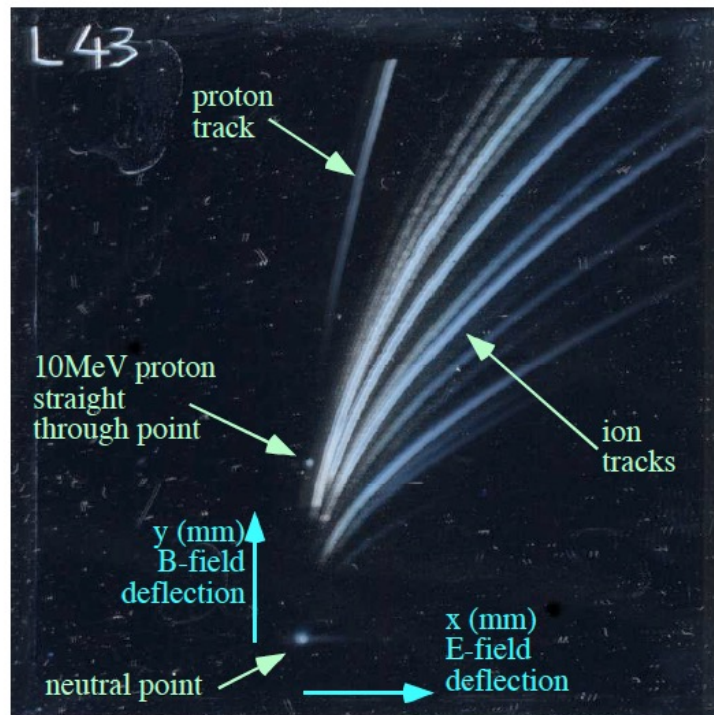
Plasma acceleration

# Acceleration of ions

## Proton and ion diagnostics: Energy spectra Thomson Parabola Spectrometer

Assuming  $v \ll c$ , the kinetic energy of the ion,  $E_{ion} = \frac{1}{2}Am_u v^2$ , is therefore:

$$E_{ion} = \frac{\left[ZeBL_B \left(\frac{1}{2}L_B + l_B\right)\right]^2}{2Am_u} \frac{1}{y^2}$$



*Courtesy of Louise Willingale, CAS 2019*  
Plasma acceleration

# Outlook

- Plasma acceleration is a new technique to accelerate particles with a high gradient.
- It is still a research topic.
- Performances have been demonstrated but beam quality still has to be improved.
- Beams are different from conventional accelerators beams.
- Some applications are been considered (FEL, isotope production,...)
- Colliders applications have been discussed but are still far away.

# More details

- You can find a large amount of courses on this topics on the website of the CERN accelerator School 2019:  
<https://indico.cern.ch/event/759579/>