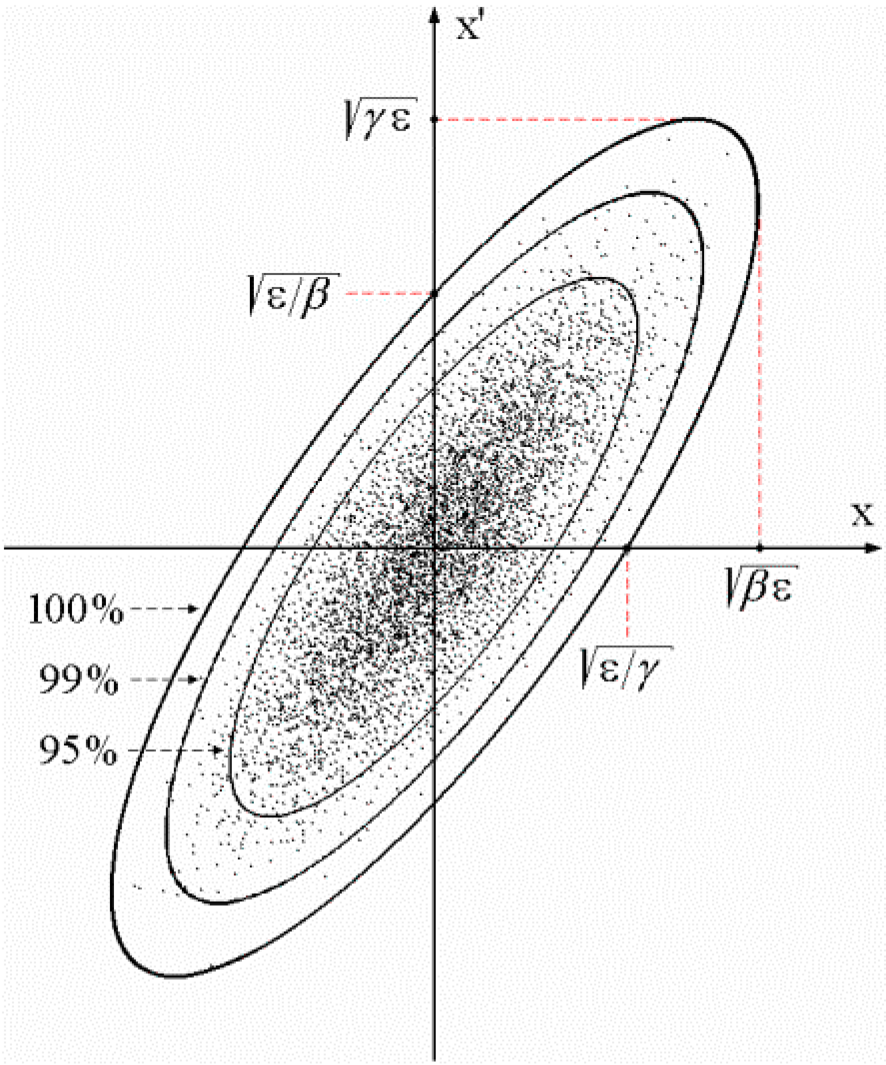
6. Emittance
(and Liouville's theorem)



- Perfect gas analogy
- Emittance
- Liouville's theorem
- Acceleration
- Radiation damping
- Emittance measurement

Problem sets 1 and 2

Next step...

You have already studied

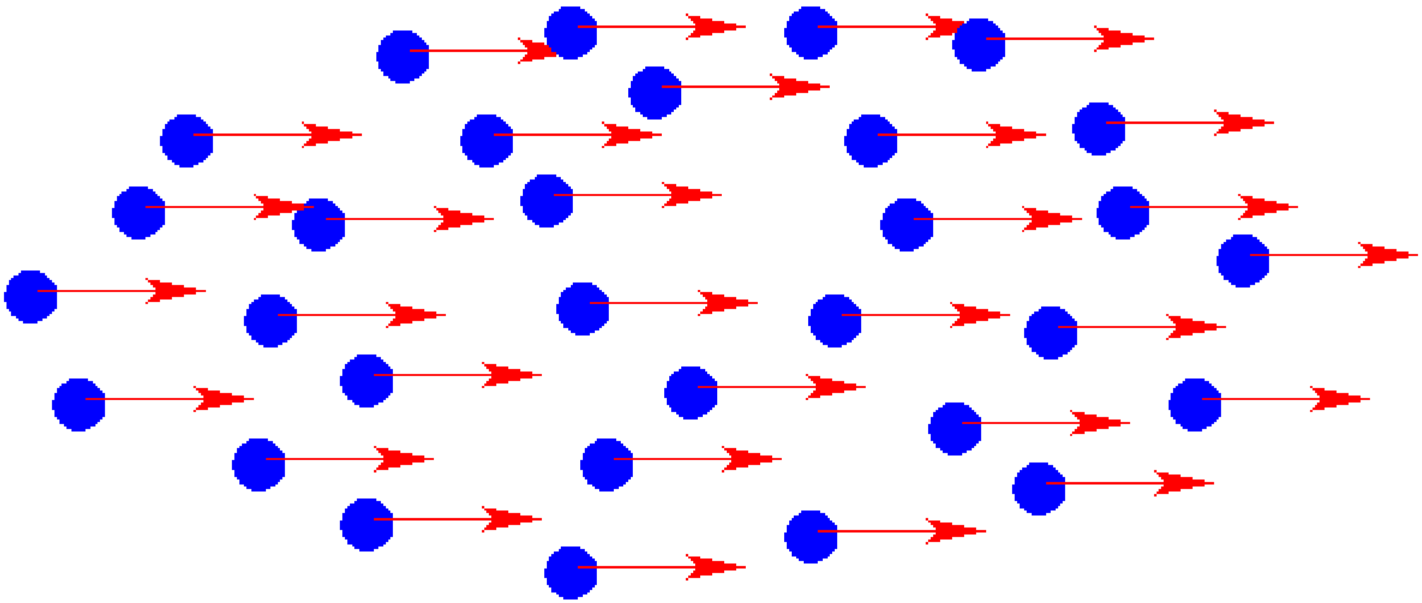
- How a particle beam is produced (lecture 2)
- How a particle beam is accelerated (lecture 3)
- How a particle beam is steered (lectures 4 and 5)

You now know all the basic blocks needed for an accelerator! That could be the end of the story but it is not...

This week we will study what happens inside the beam and how this affects the accelerator's performance.

Note: Today we will ignore the particle charge. Effect due to Coulomb repulsion (also called space-charge effects) will be studied next week (lecture 8).

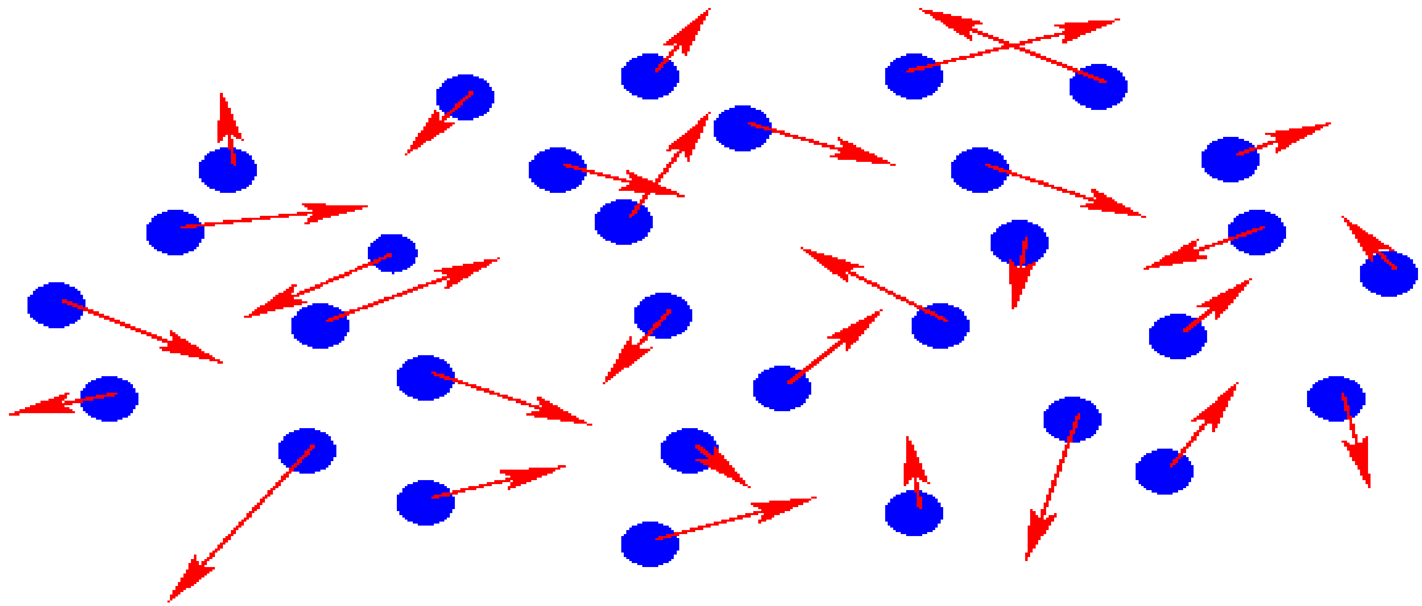
Let's look at a particle bunch



An observer in the laboratory frame looking at a particle bunch will only see particles travelling at the speed of light, apparently all in the same direction.

It is very different if one looks in the bunch's centre of mass frame...

Let's look at a particle bunch



In the centre of mass of the bunch, the particles do not look so well organised...

This should remind you other statistical systems that you have already studied: Gases!

Perfect gas law

- You have studied earlier that a perfect gas obeys the law: PV=nRT
- V is the volume term (V=xyz)
- P is a dynamic term: P, the pressure is proportional to the amount of scattering experienced by atoms as they travel in the volume. It is proportional to the momentum of the gas atoms (P~x'y'z').
- Hence it is possible to write that for gas atoms the product of their position by their momentum is expressed by their temperature *(times a constant).*
- We have seen that in the CoM particles look like a perfect gas. The product of their position by their momentum is called the “emittance” of the beam.

6D Trace space

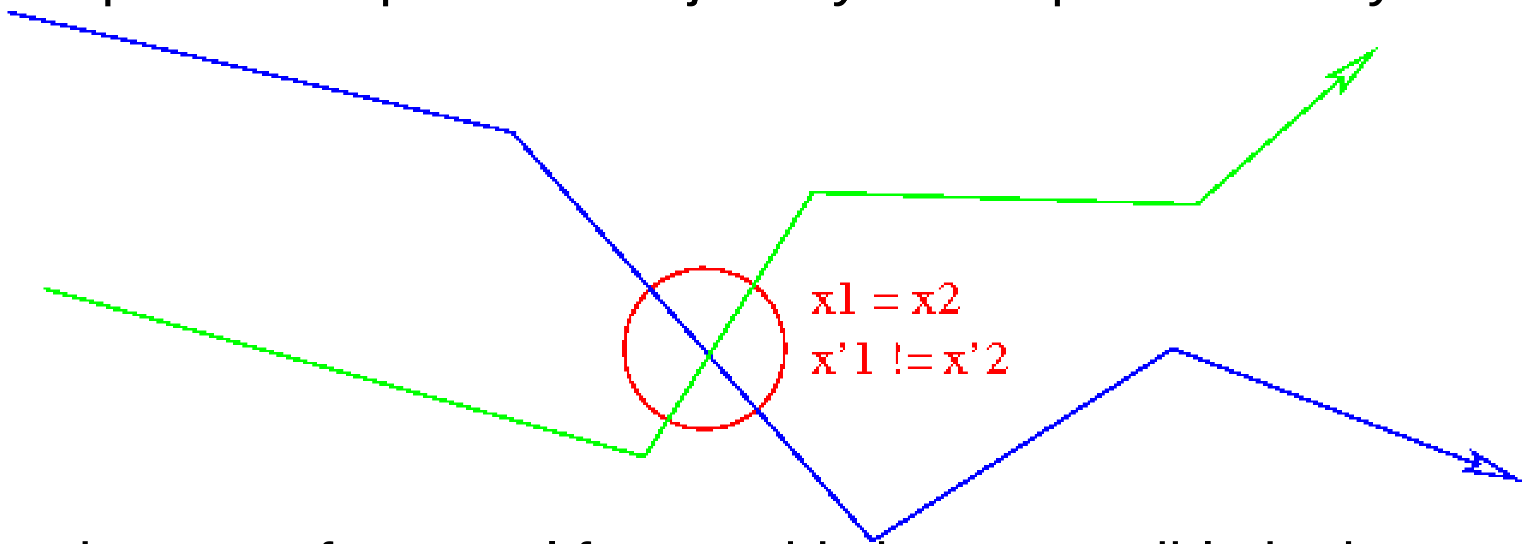
- The position-momentum 6D space is called the trace space.
- To help visualisation the trace space can be decomposed in 3 orthogonal position-momentum planes:

$$xyzx'y'z' = xx'*yy'*zz'$$

- It is also often useful to look separately at the transverse and longitudinal planes.

An interesting property of the trace-space: laminar flow

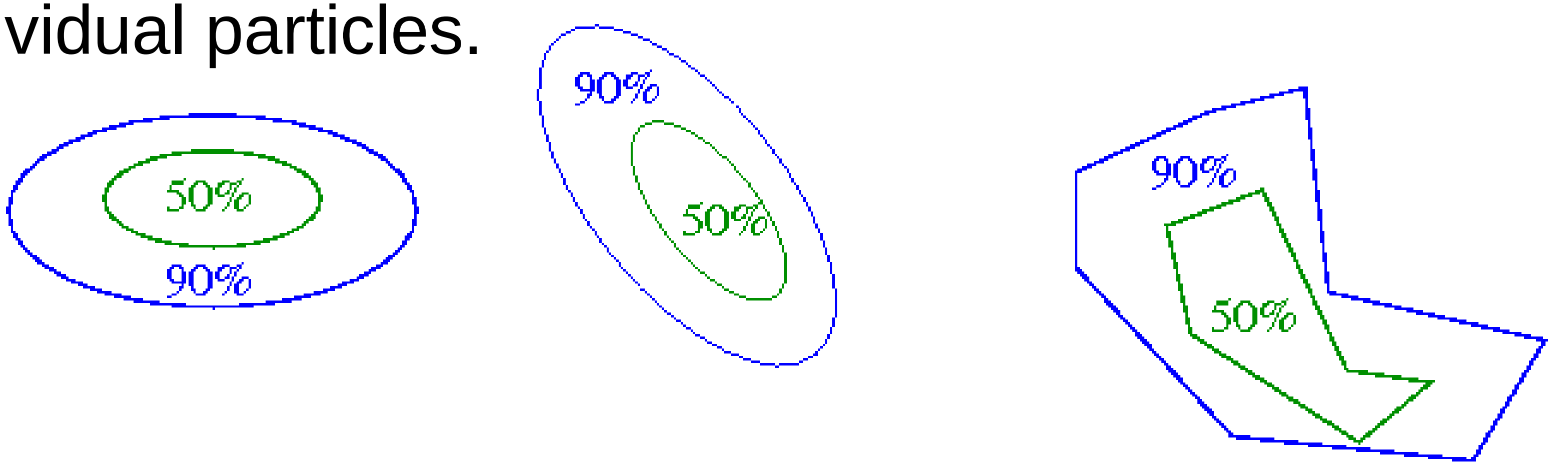
- In the 3D-positions space the trajectory of two particles may cross.



- In the absence of external forces, this is not possible in the trace space:
  - x1(t) = x2(t) and x'1=x'2 and x''=0 => x1(all t)=x2(all t)
  - If two particles are at any given time at the same position in the trace space, they follow the same trajectory!
  - These two particles have the same future (and the same past).
- This is called “laminar flow”: particles flow in separate in sheets that do not cross.

Laminar flow and beam envelope

- Laminar flow has an interesting property:
  The beam envelope at time t will the beam envelope at any time, both in the past and the future.
- In trace-space no particle can cross the beam envelope (that would violate laminar flow conditions)
- Propagating the beam envelope (or any envelope containing a given fraction of the beam) allows to see how the beam will propagate without having to study each individual particles.



Liouville's theorem

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \sum_{i=1}^{d} \left(\frac{\partial \rho}{\partial q^i} \dot{q}^i + \frac{\partial \rho}{\partial p_i} \dot{p}_i \right) = 0.$$

- The volume occupied in the phase space by a system of particles is constant.
- This is a general physics theorem, not limited to accelerators.
- The application of external forces or the emission of radiation needs to be treated carefully.

Joseph Liouville
1809-1882
(source: wikipedia)

Emittance

- We have defined the emittance as the volume occupied by the beam in the trace space.
- Liouville's theorem tells us that such volume must be constant.
- Hence the emittance of a beam is constant (unless external forces are applied).

- The total volume occupied by the bunch in trace-space is usually dominated by a few far-outlying particles.
- Instead of giving the volume occupied by all the particles, it is common to give the volume occupied by 90% or 60% of the particles or to give the RMS emittance.
- The fraction of particles included in the emittance is usually quoted.

ϵ_{90} ϵ_{RMS}

Quizz

1) Is the emittance of a particle beam an intensive or an extensive physical property?

- a) Intensive
- b) Extensive

2) In which units should the transverse emittance in 1 dimension (x,x') be expressed?

- a) Square metres
- b) Barns
- c) Metre x radians
- d) Kelvin

Answer: (1b)

- Emittance is the product of the beam size by the beam divergence.
- Both are extensive quantities
- The emittance of a fraction of a beam will be smaller than the emittance of the full beam.

$$\epsilon_{90} > \epsilon_{60}$$

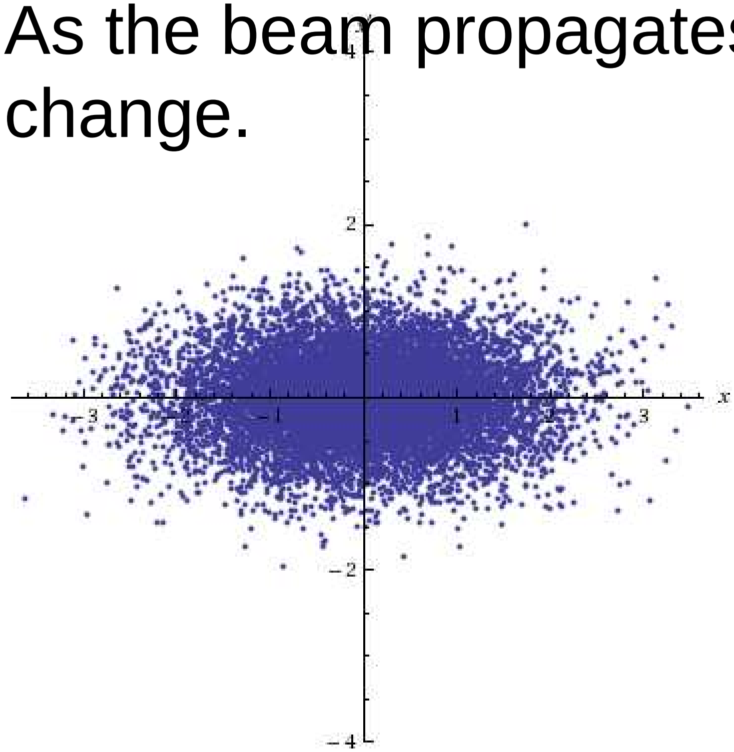
- This property is used in particle accelerators: the emittance of a beam can be improved by trimming it.

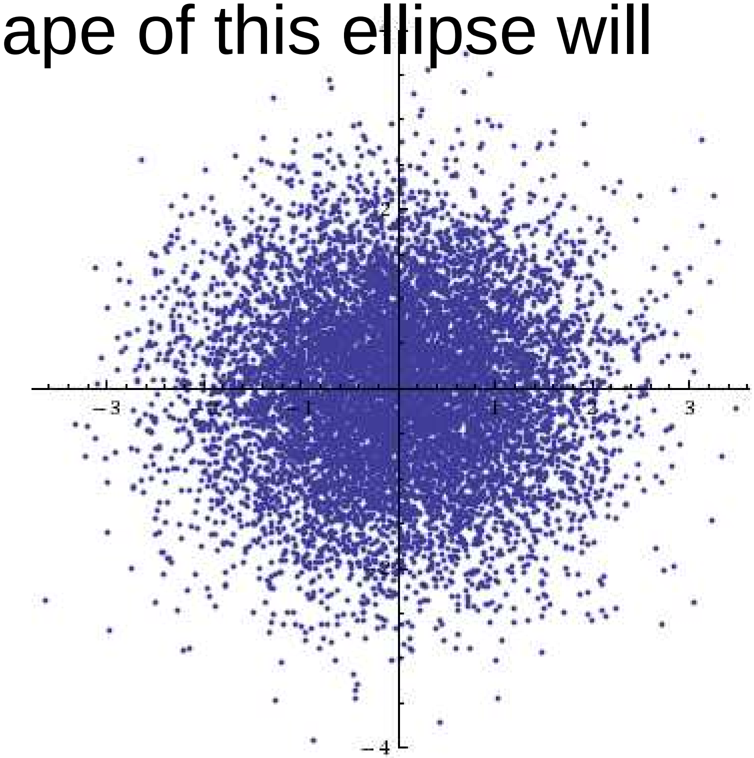
Answer: (2c)

- The size of the beam is expressed in (milli)metres.
- The divergence of the beam can be expressed either in (milli)radians or in eV.
- Depending on conventions a factor Pi may be added.
- A typical emittance for an electron linac with a thermionic gun is mm.mrad.
- A synchrotron can go below nm.mrad.
- Do not forget that 1 mm.mrad = 1um.rad!

Emittance ellipse

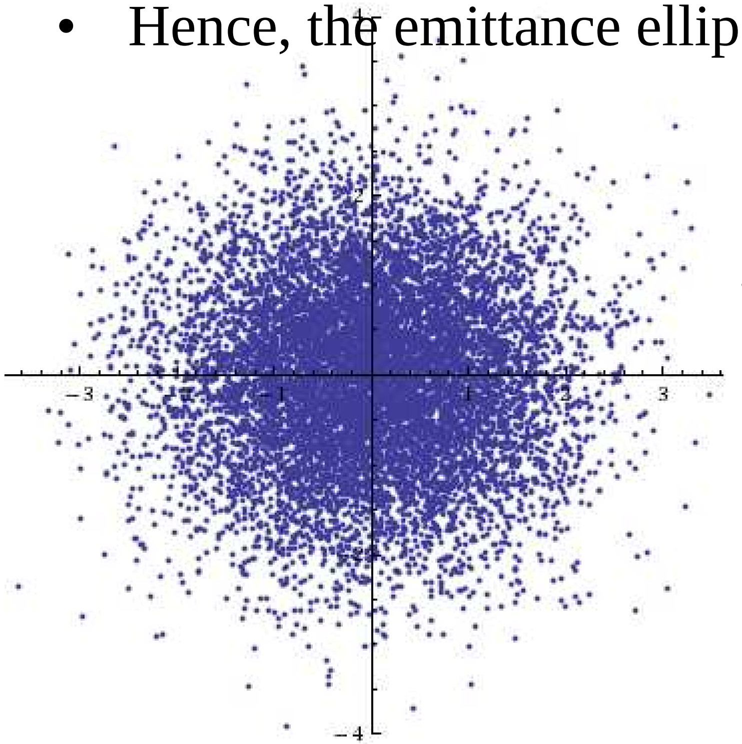
- A random gaussian distribution of particles forms a straight ellipse.
- By choosing the right set of coordinates this ellipse can be transformed in a circle (do not forget that the two axis are orthogonal!).
- As the beam propagates, the shape of this ellipse will change.



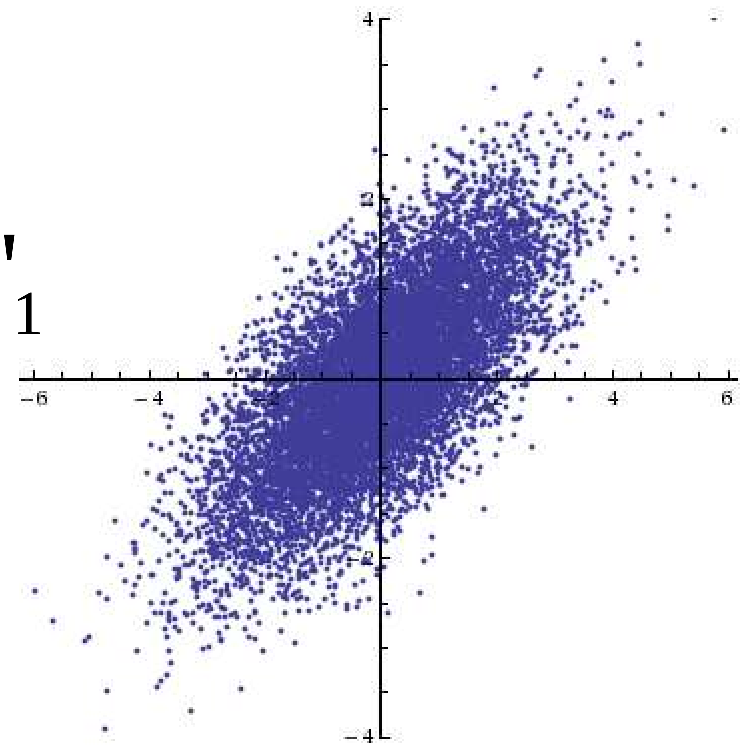


Beam drift

- When the beam "drifts" that is, propagates in space, over a length L with no external forces applied:
  - The momentum of the particles is constant
  - The position changes by the momentum times L.
- Hence, the emittance ellipse is sheared.

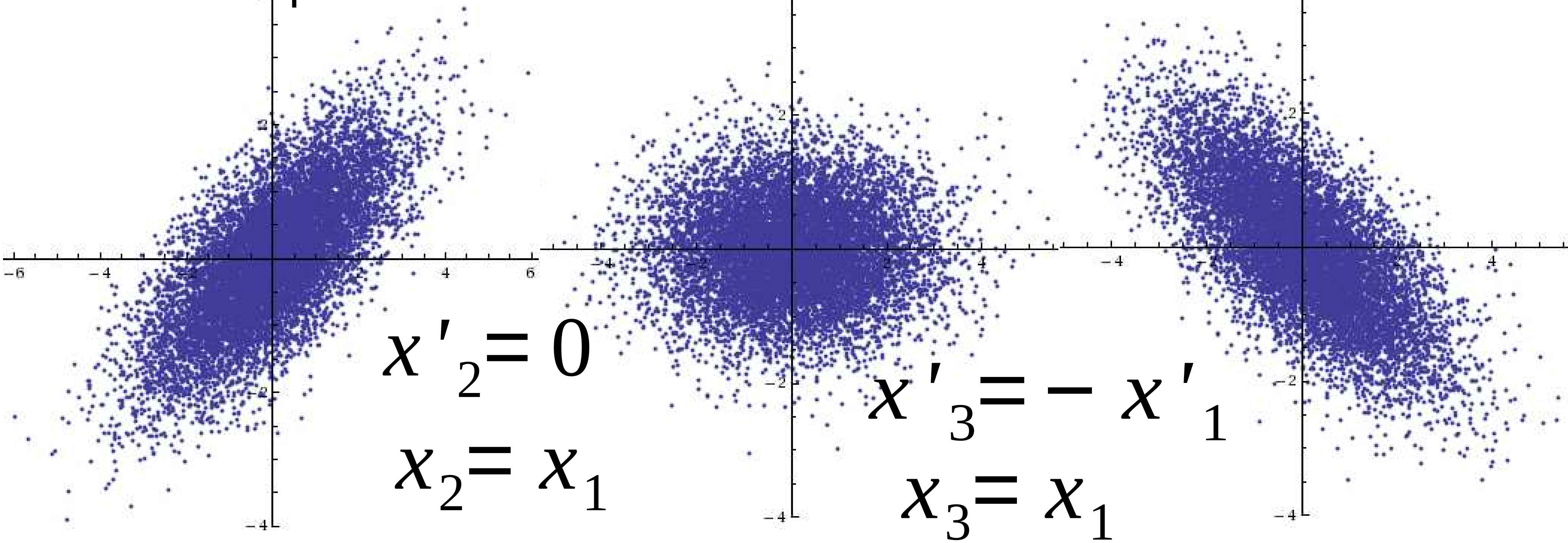


$$x'_2 = x'_1$$
$$x_2 = x_1 + Lx'_1$$



Focussing

- In a focussing section (typically a quadrupole), in the thin lens approximation:
  - The position of the particles is not affected
  - The momentums are reversed, hence a waist (at which all x'=0) is formed.
- The ellipse is unsheared and then sheared in the other direction

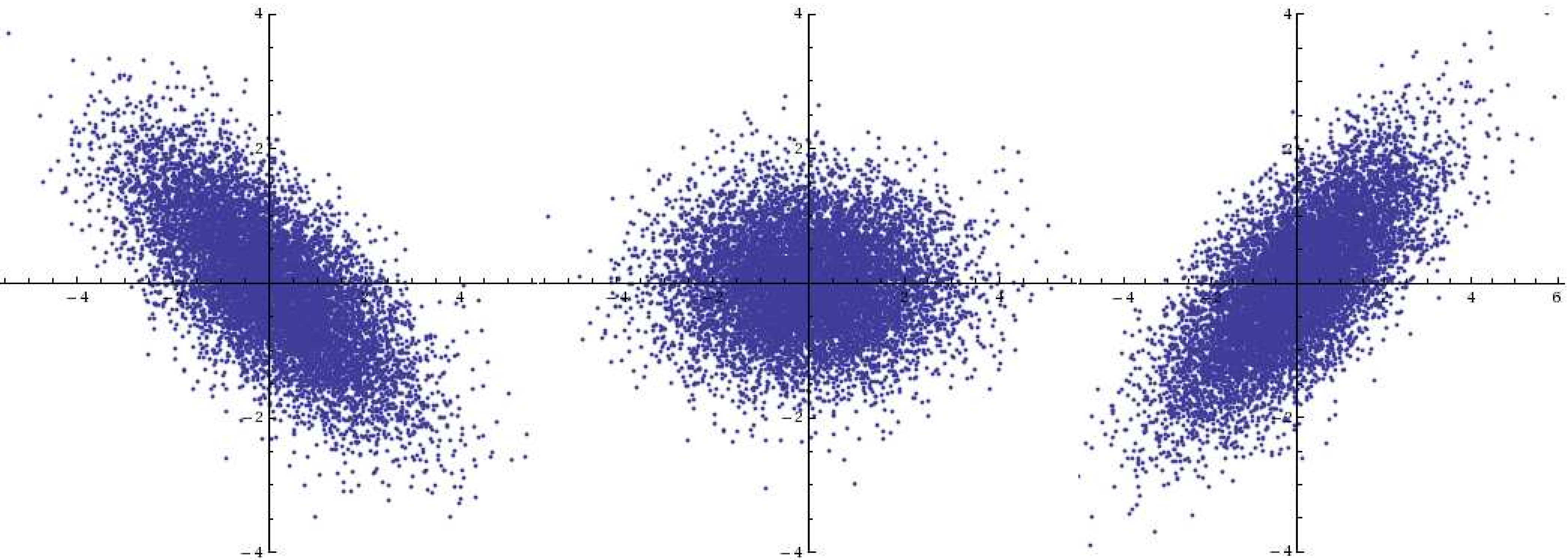


$$x'_2 = 0$$
$$x_2 = x_1$$

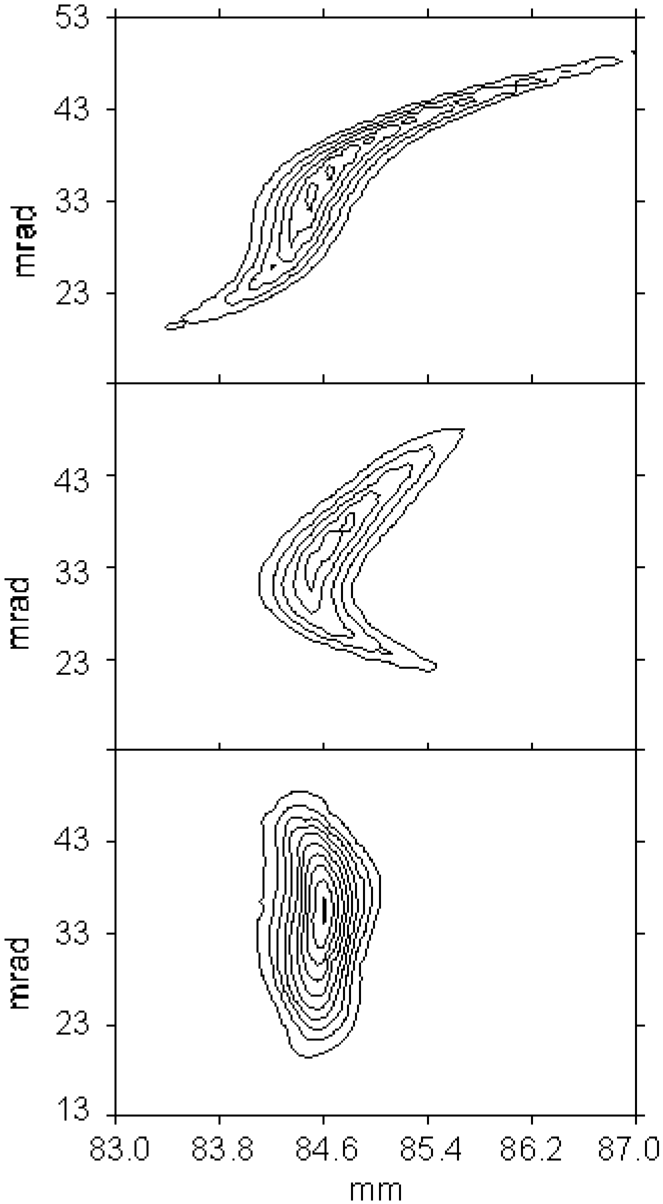
$$x'_3 = -x'_1$$
$$x_3 = x_1$$

Beam waist

- After the focussing section the beam will drift again, decreasing the shearing of the emittance ellipse.
- At some point the momentums will again average to 0, the beam will be forming a waist.
- At the waist the shearing of the emittance ellipse flips and starts increasing again.



Non linear effects



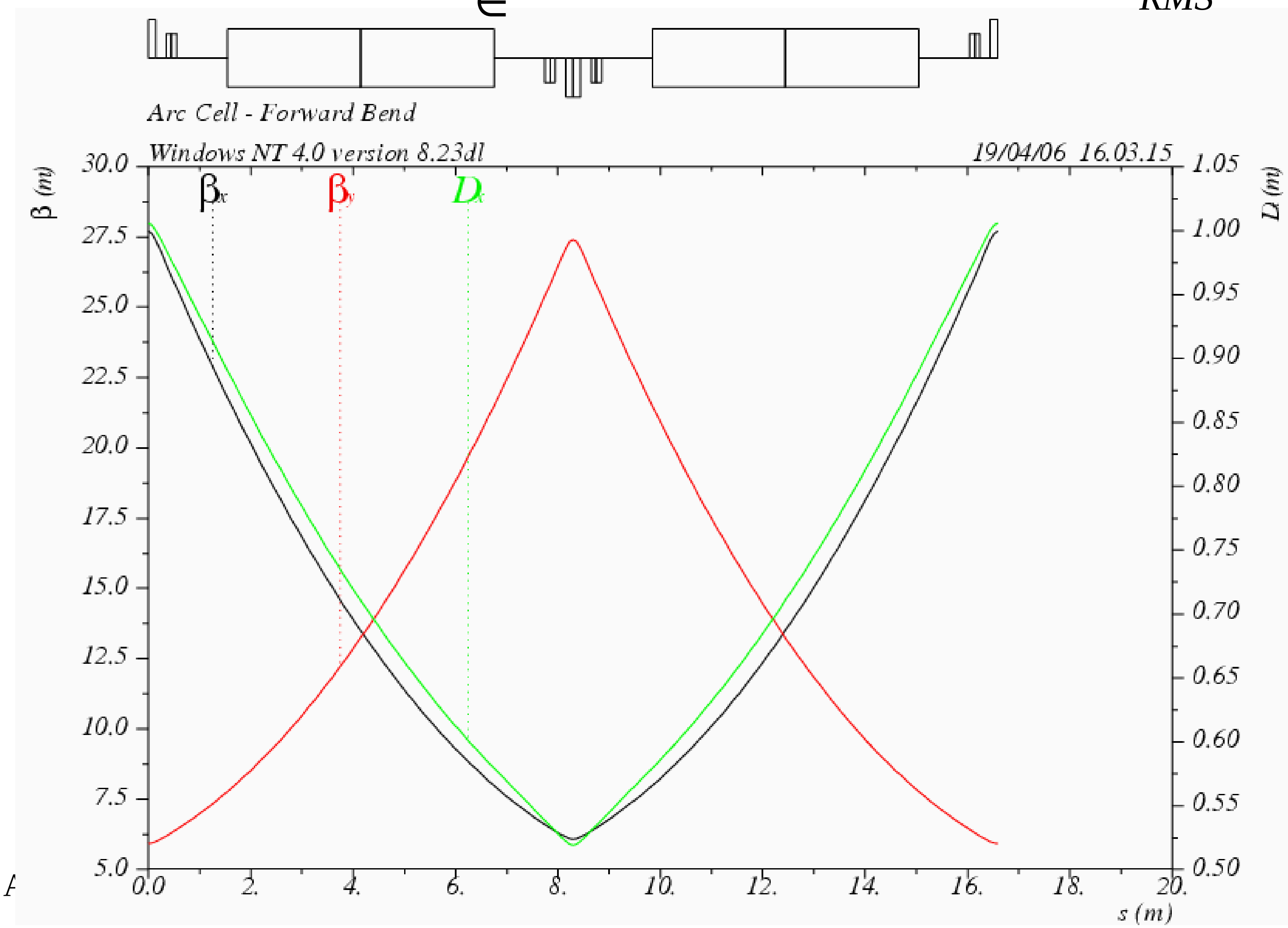
Source:TRIUMF

- Magnet non linearities will increase the deformation of the emittance ellipse.
- Higher order magnets are required to correct such deformations (octupole and sextupole).
- Example of emittance measured at an accelerator in Canada.

Beta function

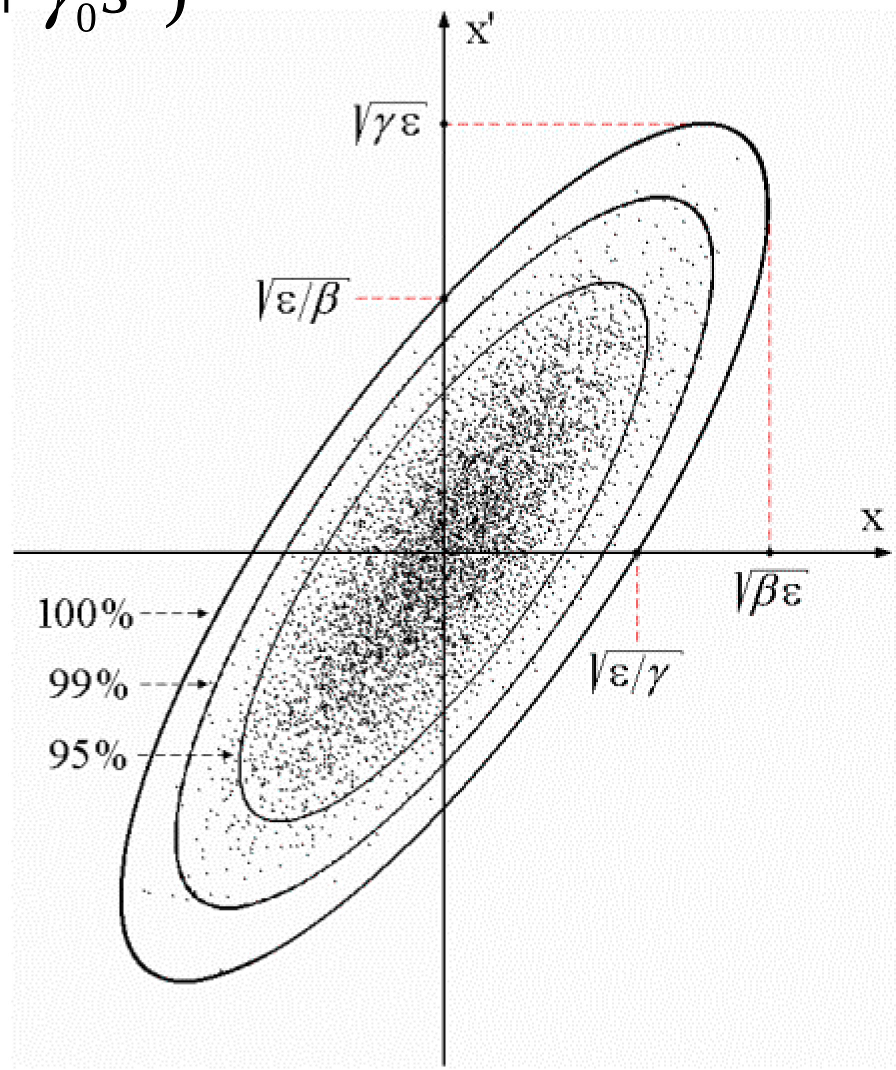
It is convenient to define the "beta function" to relate the beam size to the emittance.

$$\beta = \frac{\sigma^2}{\in} \qquad \sigma = \sqrt{\beta \in} \qquad \sigma_{RMS} = \sqrt{\beta \in_{RMS}}$$



Beam parametrisation: Twiss parameters

$$\sigma = \sqrt{\in (\beta_0 - 2\alpha_0 s + \gamma_0 s^2)}$$

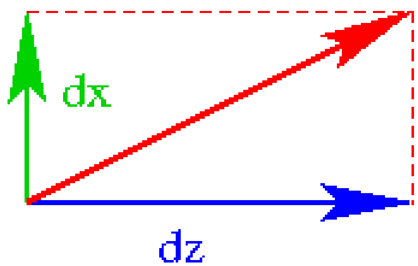


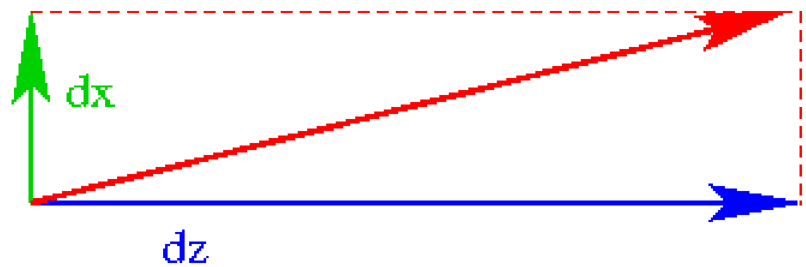
- The emittance ellipse can be described using the Courant-Snyder representation (Twiss parameters).
- Like any other physical system in which elements travels in straight line, the beam envelope forms an hyperbola.

$$\sigma = \sqrt{\in (\beta_0 + \frac{s_w^{\ 2}}{\beta_0})}$$

Acceleration

- When the beam is accelerated, its longitudinal momentum is increased,
- But the transverse momentum remains the same.
- Hence the beam divergence decreases.





- Accelerating the beam leads to a reduction the volume occupied in phase space.
- This reduction is proportional to the increase of the relativistic gamma.

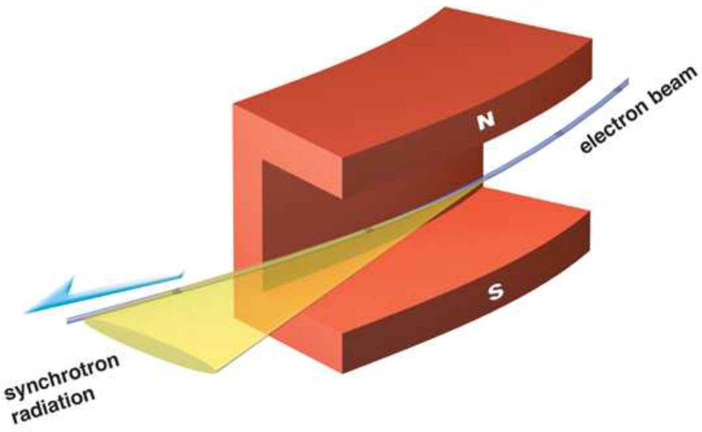
Normalised emittance

- It is convenient to define the normalised emittance of a beam: it is the volume of phase space occupied by the beam multiplied by gamma.
- The actual volume of phase space occupied by the beam is called the geometric emittance.
- The normalised emittance of a beam is constant under acceleration.

$$\in_N = \gamma \in_{Geometric}$$

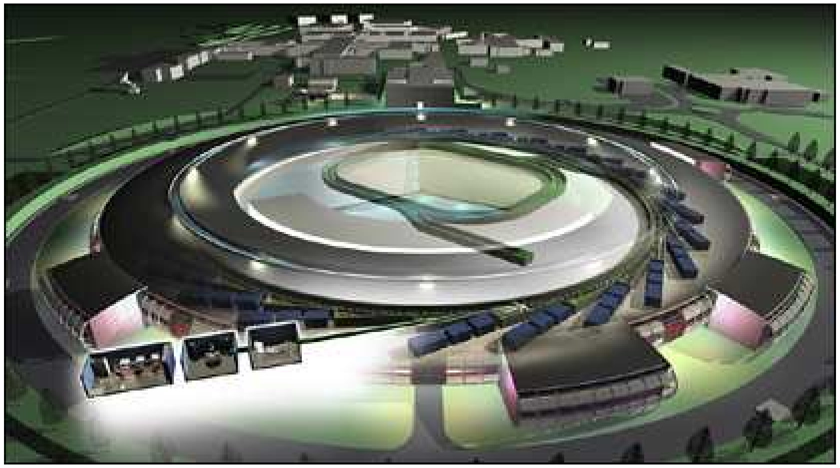
Radiation damping (1)

- In a synchrotron particles emits synchrotron radiation.
- This emission is compensated by a RF cavity that tops-up the energy of the beam at each turn.
- This additional acceleration at each turns results in a decrease of the beam emittance.
- By storing a beam in a ring for several milliseconds it is possible to significantly reduce its transverse emittance.



Radiation damping (2)

- The reduction of emittance in a ring due to SR emission is called “radiation damping” and the ring used for such purpose are called damping rings. The time required to achieve such damping is called the damping time.
- As the radiation is emitted in the plane of the accelerator, radiation damping is faster in the direction orthogonal to the accelerator.
- Synchrotron (and especially 3rd generation light sources) use radiation damping to operate at very low emittance.



Source: Diamond

Emittance coupling

- When the beam is deflected by a magnet (dipole or quadrupole) the bending of the beam depends on its total momentum (px+py+pz).
- Particles with different momentum will be bent differently.
- In a bending magnet there is a coupling between the longitudinal and transverse emittance.
- As the longitudinal emittance is usually bigger than the transverse emittance this will result in an increase of the transverse emittance.
- Coupling also occurs between the two transverse directions.
- In a damping ring the bending magnets are the first source of coupling. Hence the transverse emittance in the plane of the accelerator will be larger than the transverse emittance orthogonal to that plane
  => after a damping ring beam are usually “flat”.

Alignment issues

- In an accelerator, the misalignment of a device will results in a residual dipole field on the beam axis.
- Such residual dipole field create a transverse deflection and hence an emittance increase.
- To avoid this all the accelerating cavities and magnets must be aligned very carefully with a precision of a few micrometres and re-aligned every few months.
- A bad quality accelerator vacuum may lead to emittance growth.

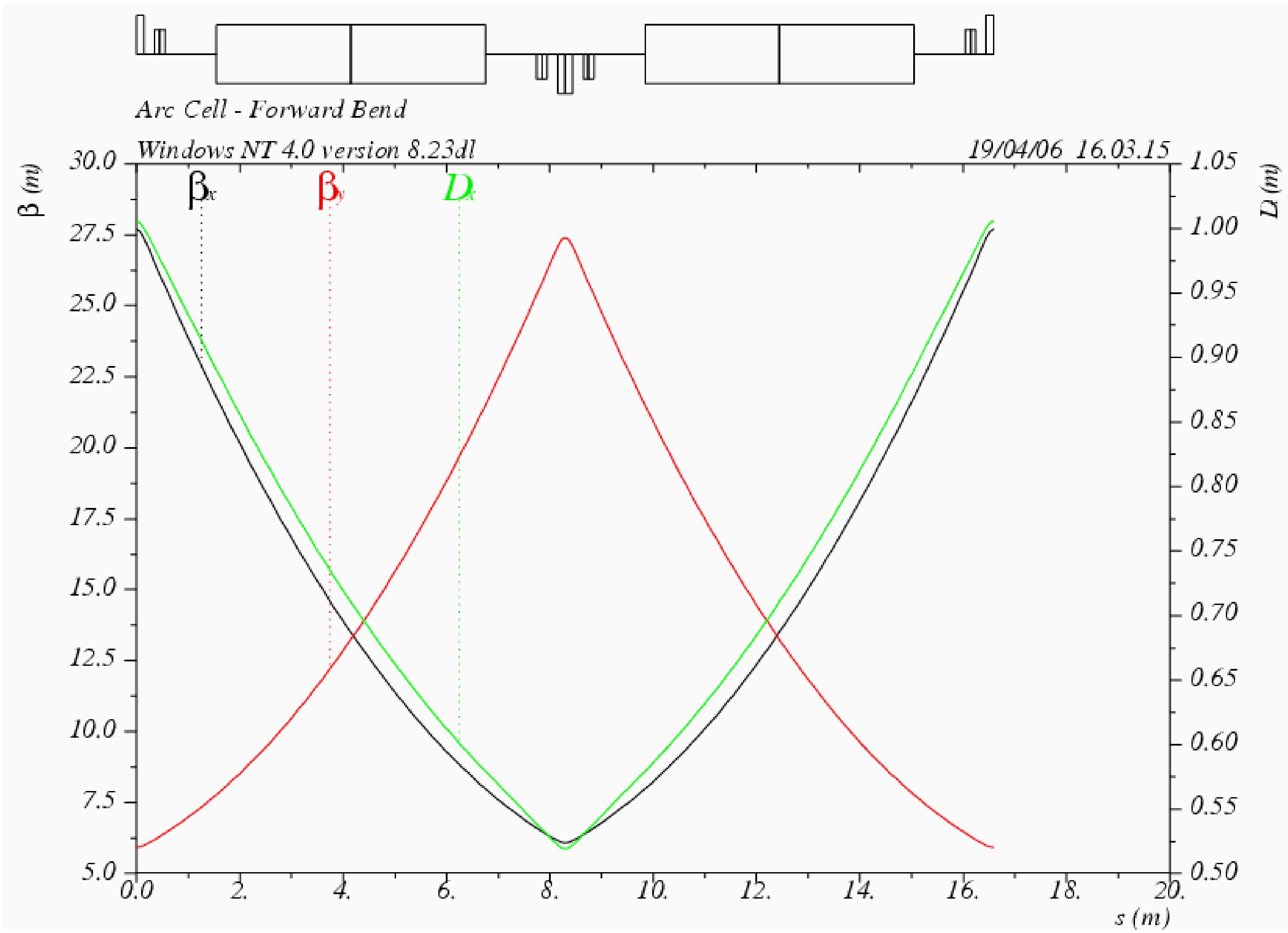
Lattice and emittance

- Ring lattices in which the beam has a large beta function or a large chromaticity in the bending magnets will lead to strong emittance coupling.
- The minimum emittance that can be achieved is given by

$$\varepsilon_{x,\min} = \frac{F}{12\sqrt{15}} C_q \gamma^2 \frac{\theta^3}{J_x}$$

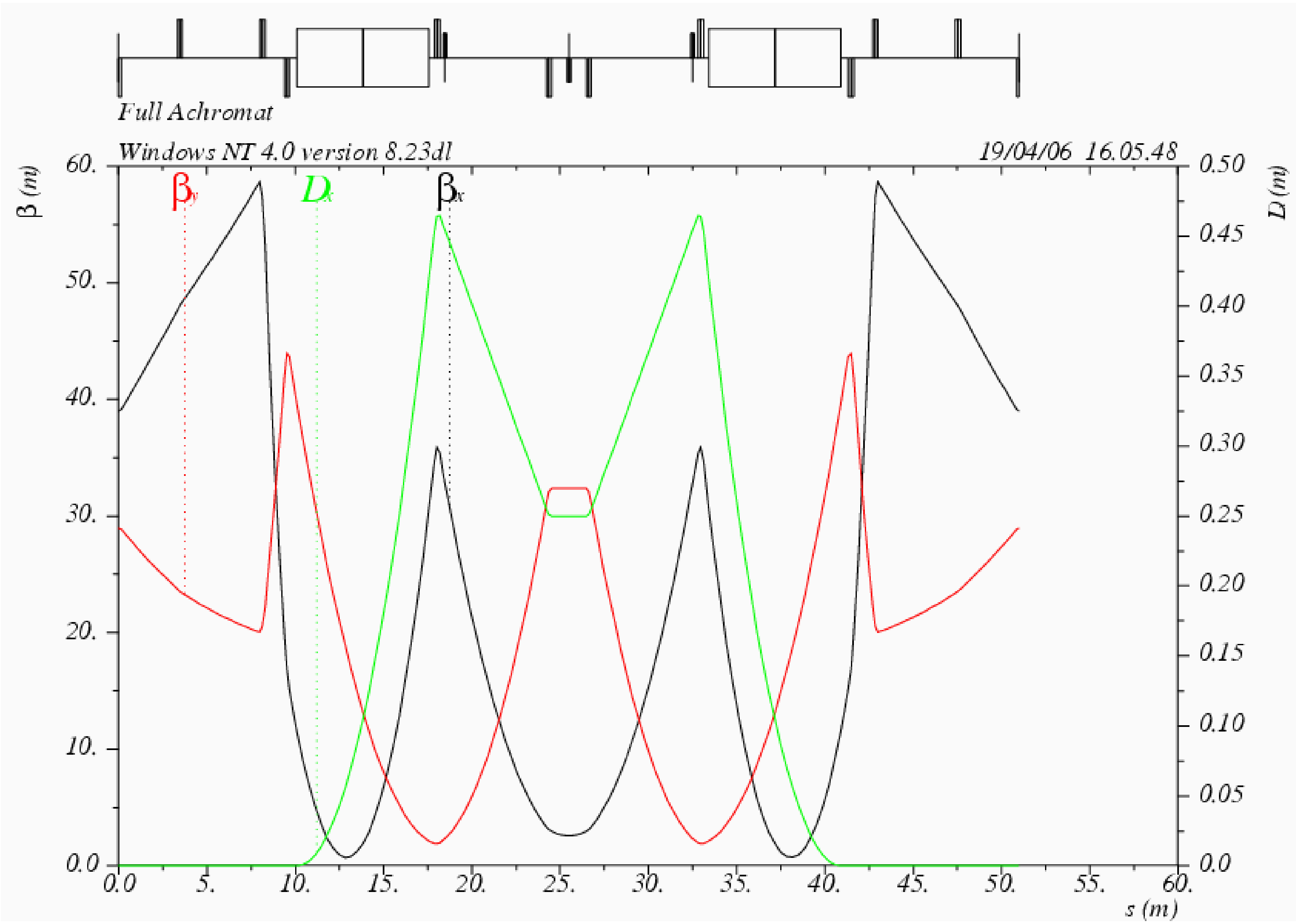
  - Where F is a factor depending on the lattice, theta the bending angle of each dipole, $C_q = 3.832\times10^{-19}$ m is a physical constant and Jx is the damping partition number.

FODO Lattice: $F \approx 100$



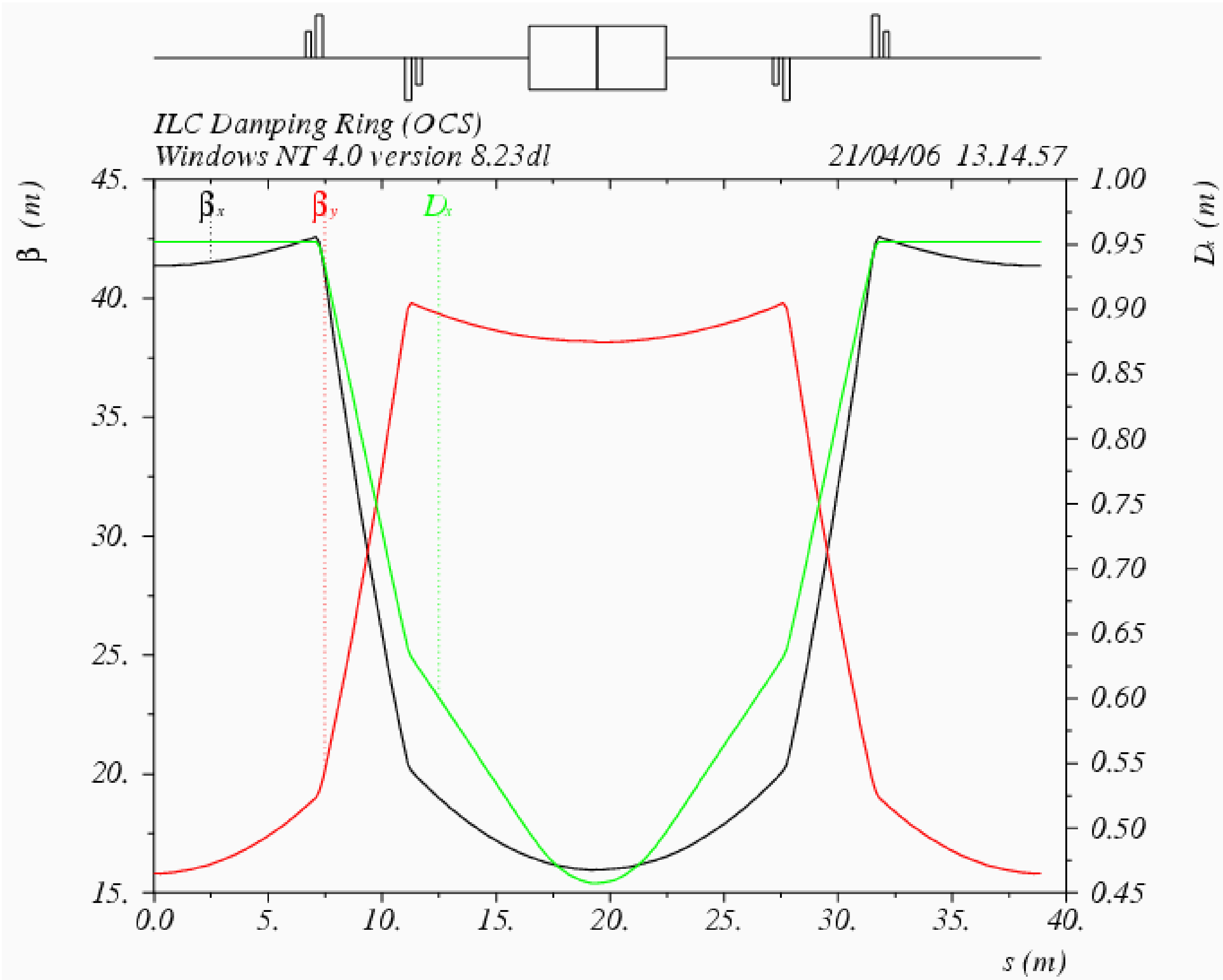
Source: Andy Wolski, ILC school

Double Bend Achromat (DBA) Lattice: F = 3



Source: Andy Wolski, ILC school

Theoretical Minimum Emittance (TME) Lattice: $F = 1$

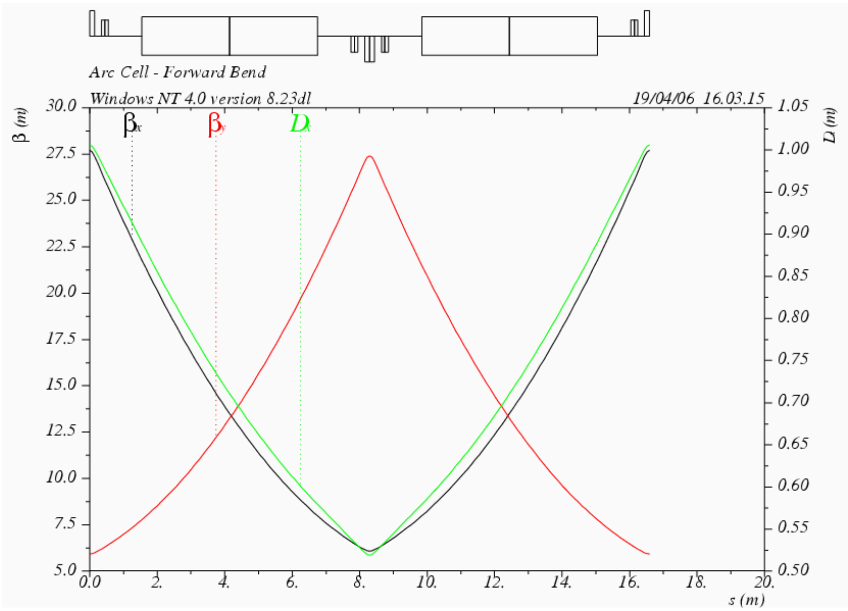


Source: Andy Wolski, ILC school

Emittance measurement: Multi screen/wire method

- The emittance is not directly an observable.
- The beam size is an observable.
- By measuring the beam size at several locations it is possible to fit the best emittance.
- The beam size can be measured by using screens or wires (beam size measurements will be discussed next week during the diagnostics lecture).

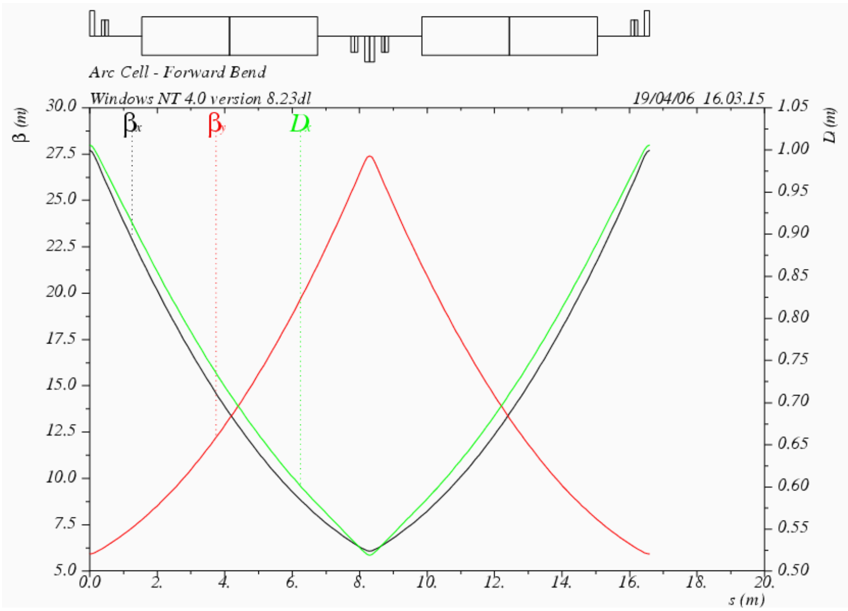
$$\sigma = \sqrt{\in (\beta_0 - 2\alpha_0 s + \gamma_0 s^2)}$$



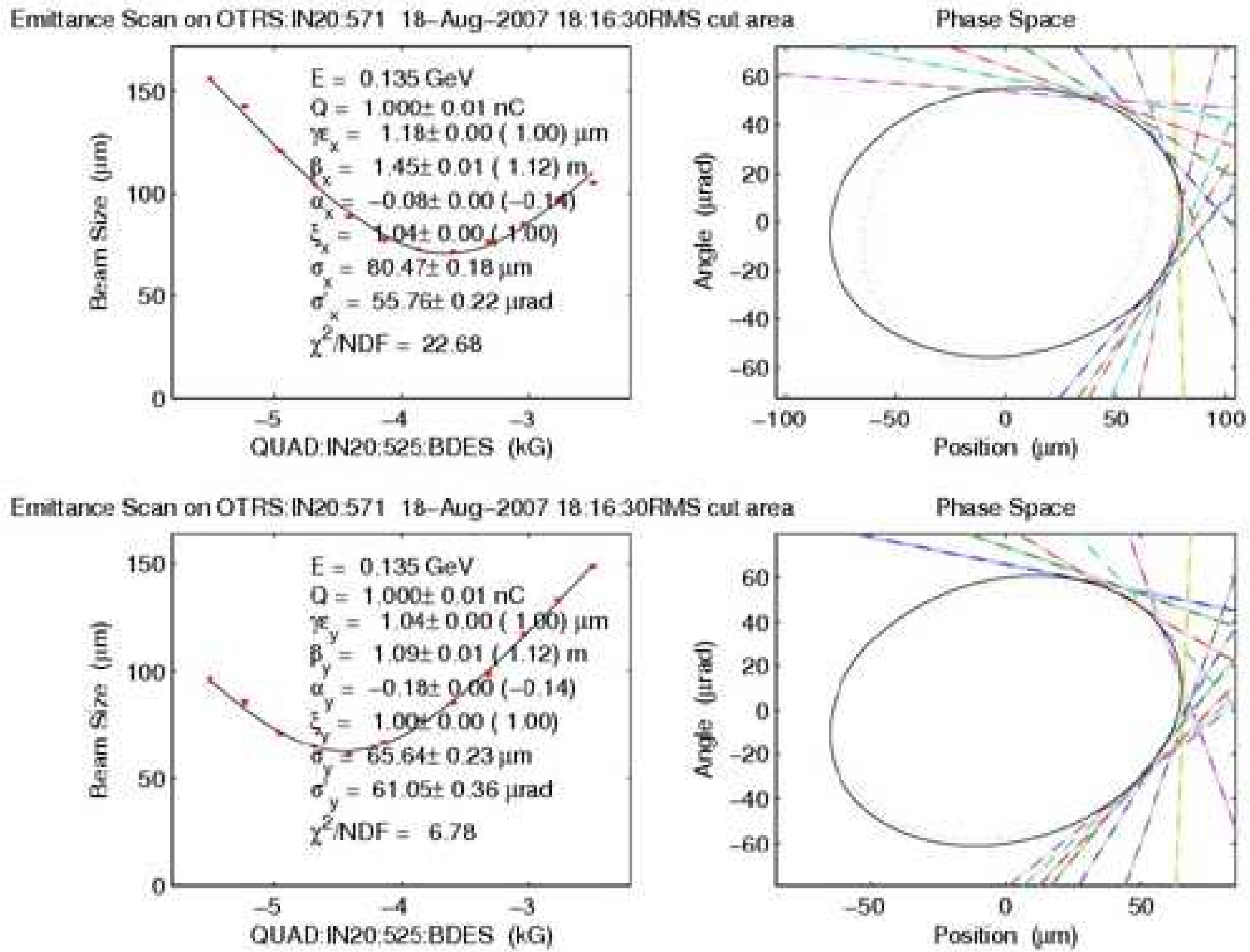
Quad scan

- The emittance can also be measured by changing the strength of a quadrupole and measuring the location at a fixed position.
- This modifies the beta function of the beam and once again this can be fitted to find the best emittance value.

$$\sigma = \sqrt{\in (\beta_0 - 2\alpha_0 s + \gamma_0 s^2)}$$



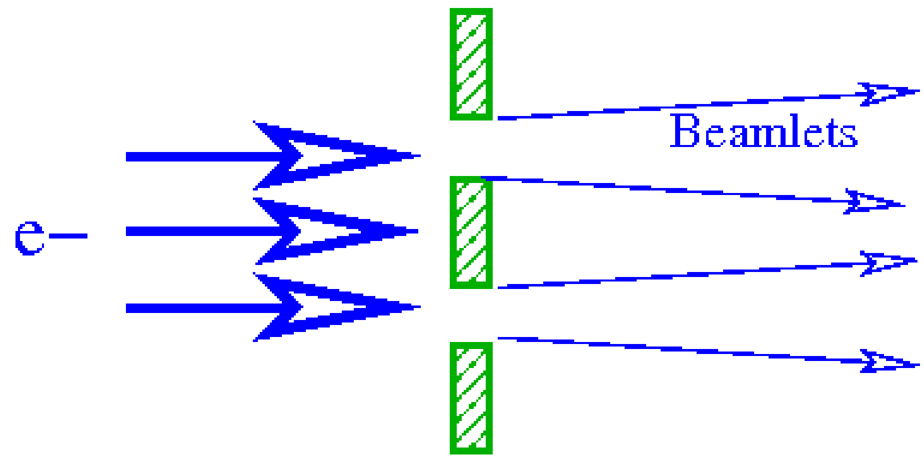
Quad scan emittance measurement

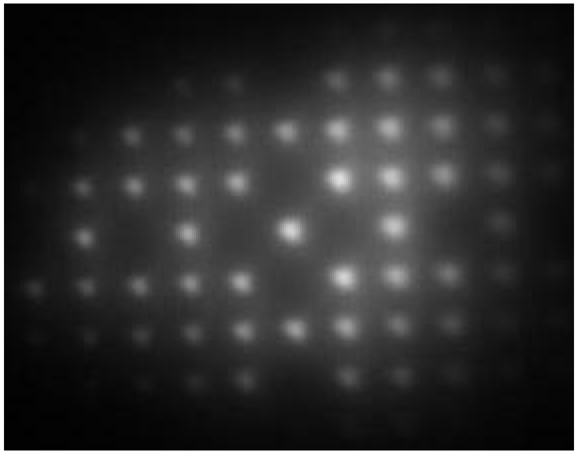


Source: LCLS

Pepper-pot (1)

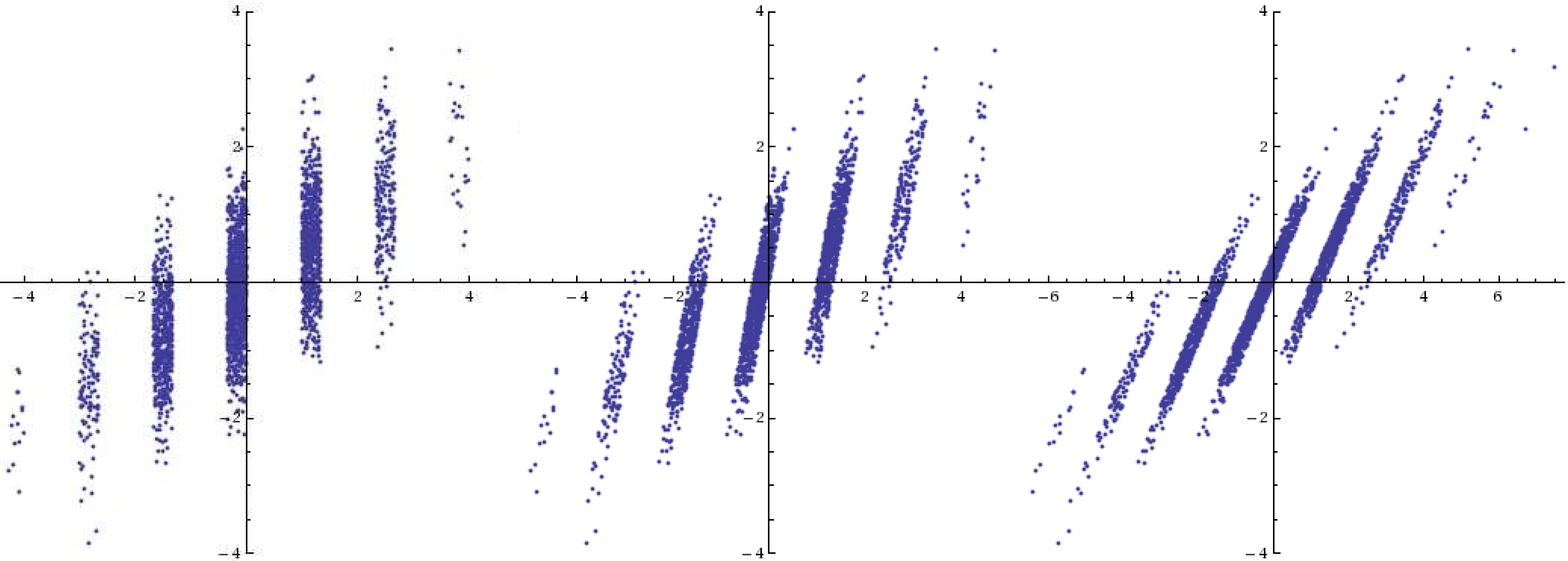
- A grid of dense material inserted in the beam path will split the beam in several beamlets.
- The transverse position at which these beamlets were created is know (it is the position of the grid).
- A measurement of the size of the beamlets downstream gives access to the beam divergence.
- The beam size plus the beam divergence can be combined to give the value of the emittance.





Pepper-pot measurement of the transverse emittance of the teaching accelerator (2007)

Pepper-pot (2)



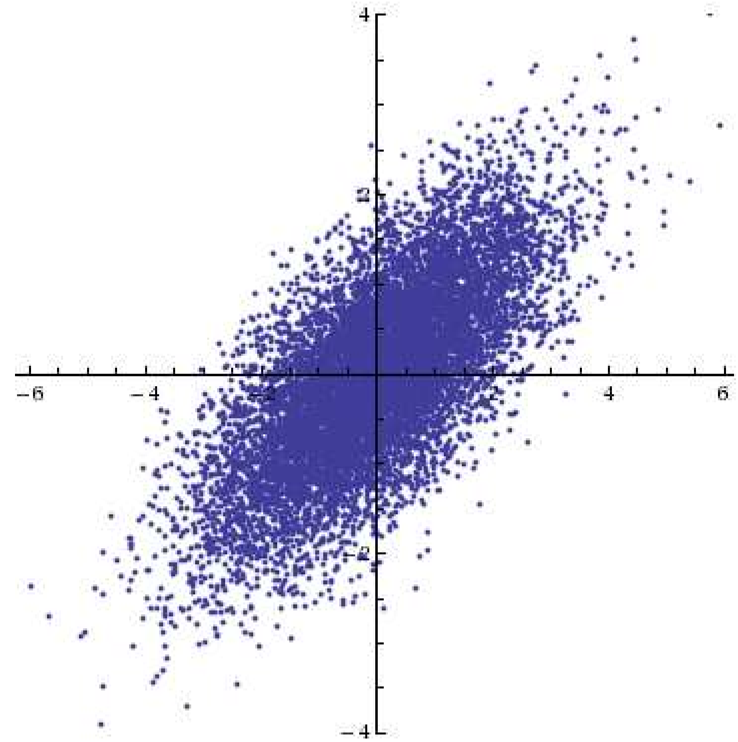
- In the phase space, the effect of pepper-pot is shown above:
  - The beam is sampled at given x positions
  - After the pepper-pot, the beam drifts
  - The measurement must be made close enough so that the beamlets do not overlap.

Pepper-pot (3)

- The Pepper-pot method is a destructive single-shot technique (the beam is destroyed after the measurement but a single pulse is enough to make the measurement).
- It is used a low energy, for example for the study of particle gun properties.
- Research is ongoing in Oxford to extend this technique to higher energies *(Delerue, Urner et al, May 2009)*

... Emittance

- Emittance is an important property of particle accelerators.
- To reach high luminosity or high brilliance accelerators need a low emittance.
- Preserving a low emittance from the source to the end can be very challenging.
- Correcting emittance distortion is sometimes required to achieve better performances.
- It is not directly possible to make a measurement in the phase space but indirect measurements can be made.



Teaching accelerator tour, Diamond visit and Problem set

- If you would like to visit the teaching accelerator, please let me know and I will arrange a tour.
- If you would like to visit Diamond Light Source, please send me an email (I need your name).
- The 6th problem set is available online.