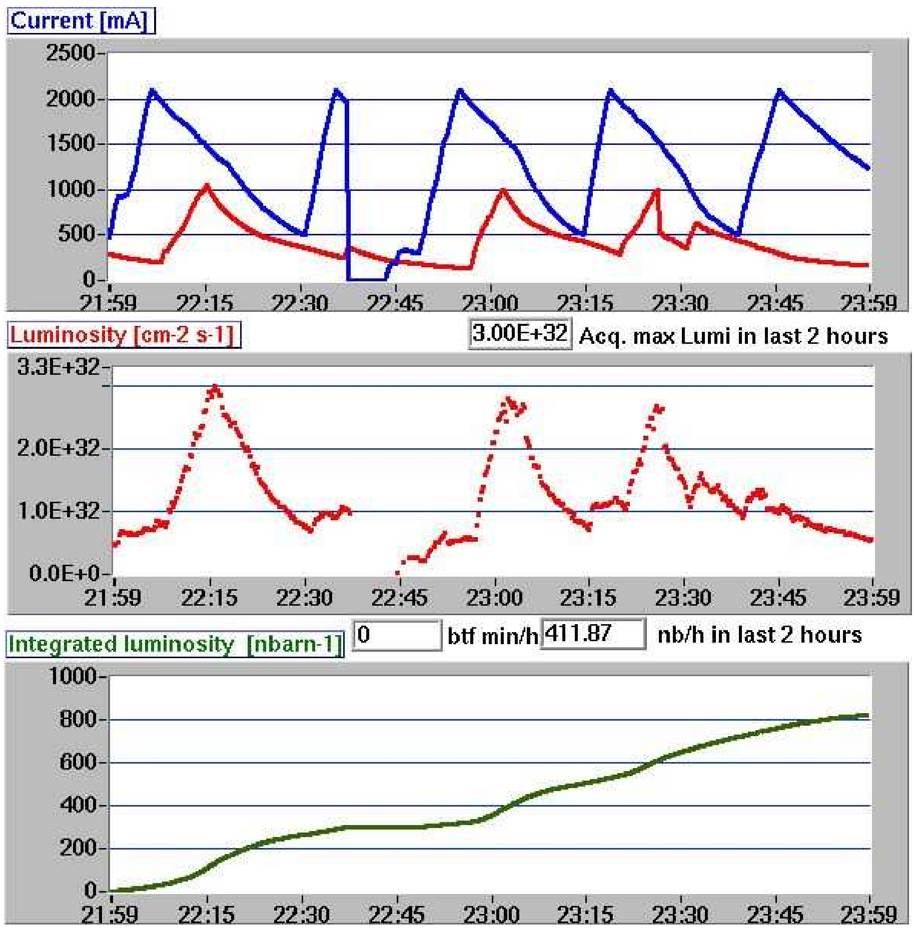
8. Space charge and lifetime



- Space charge
- Magnet apertures
- Beam lifetime

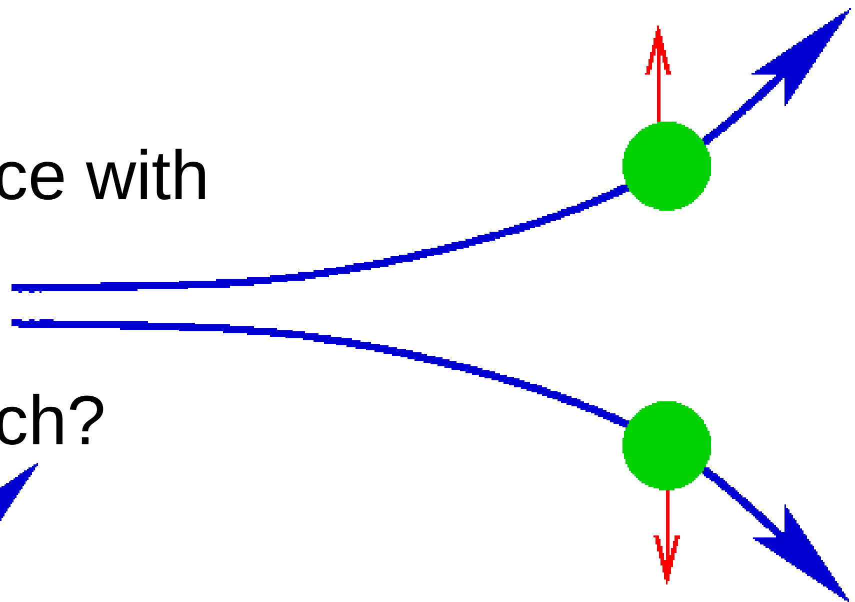
Problem set 6

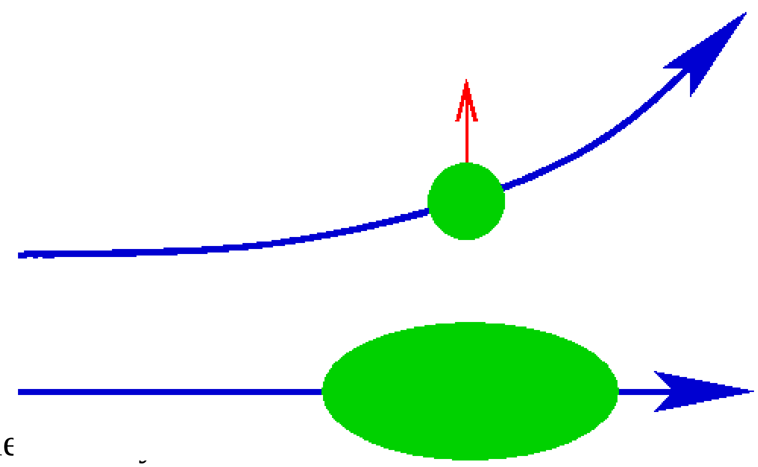
Where are we?

- Today we are going to study how the bunch affect itself and how this reduces the quality of the beam.
- We will also try to understand why beams do not have an infinite lifetime.

Space-charge effect

- Let's consider two particles with similar charges travelling in the same direction.
- Due to their charge these particles will push each other away (Coulomb's law).
- What is the intensity of the force with which they repel each other?
- What is the effect of a full bunch?

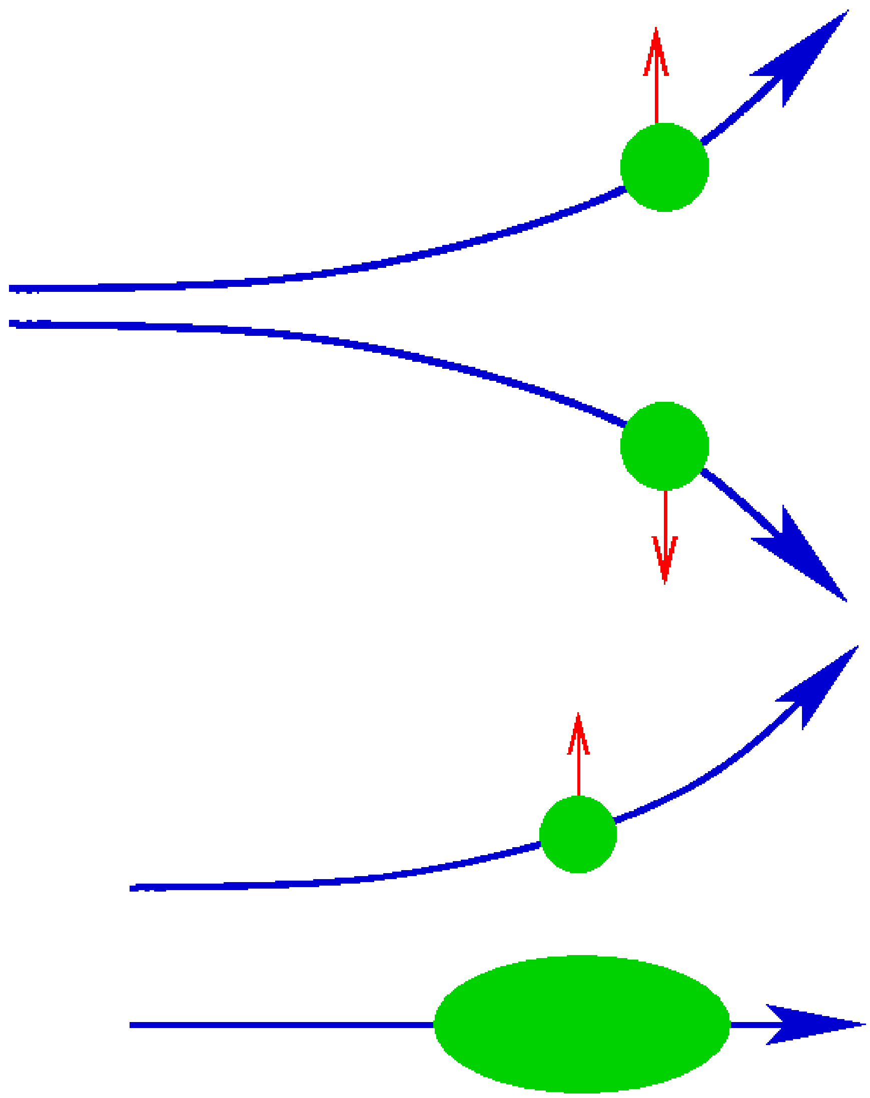




Coulomb force between two electrons

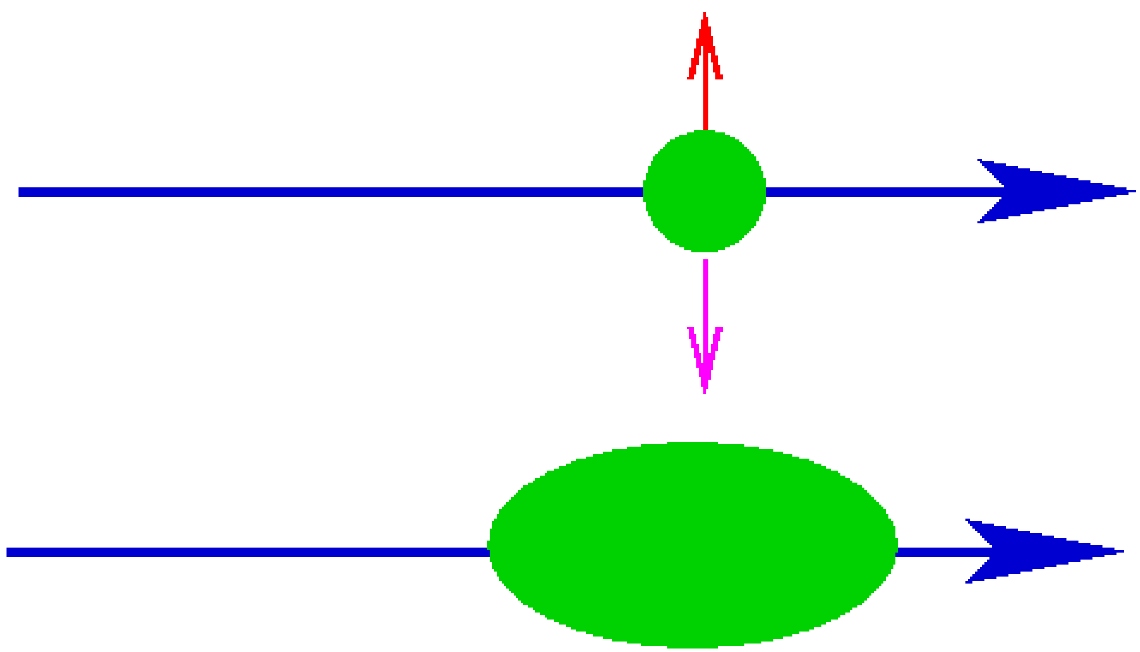
$$f = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{d^2}$$

- Assume d=1micrometre.
- $f=2\ 10^{-16}$N
- This may look small but an electron is not very heavy
- $f/m=2.5\ 10^{14}$N/kg
- This force is very intense on the scale of the electrons.
- Typical charge in a bunch: ~100pC = $6\ 10^{8}$ electrons



Avoiding space-charge effects

- If there is a second force that cancels the effect of the space-charge the particle will not be deviated.
- We have seen in lecture 2 that near the gun a compensating electrode is used.
- Let's see how the shape of this electrode is defined...



Electrostatic potential in the beam

- Assume steady state $\frac{\partial \rho}{\partial t} = 0$
- Particle conservation as they propagate
  => Current constant across gap

$$\frac{\partial [Zen(z)v_z(z)]}{\partial z} = 0$$

$$n(z) = \frac{j_0}{Zev_z(z)}$$

- Electrostatic potential is 0 at source hence, particles are accelerated in the gap.

$$\frac{m_0 v^2{}_z}{2} = -Ze\phi$$

- Hence, by substitution

$$\frac{d^2\phi}{dz^2} = \frac{-j_0}{\epsilon_0 \sqrt{-2Ze\phi / m_0}}$$

- And thus

(see also Humphries, CPB, sec 5.2)

$$\phi(z) = V_0 (\frac{z}{d})^{4/3}$$

Electrode design

- Electrostatic potential in the beam:

$$\phi(z) = V_0 (\frac{z}{d})^{4/3}$$

Laplace equation:

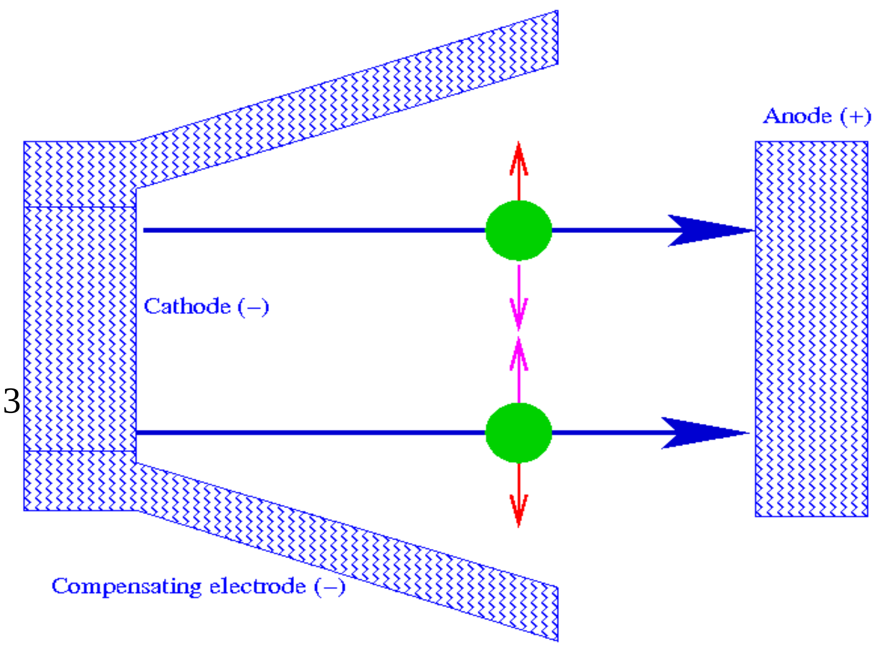
$$\frac{\partial^2 f}{dz^2} + \frac{\partial^2 f}{dx^2} = 0$$

- Trial function:

$$u = z + jx \qquad f(u) = V_0 (\frac{u}{d})^{4/3} = V_0 (\frac{z + jx}{d})^{4/3}$$

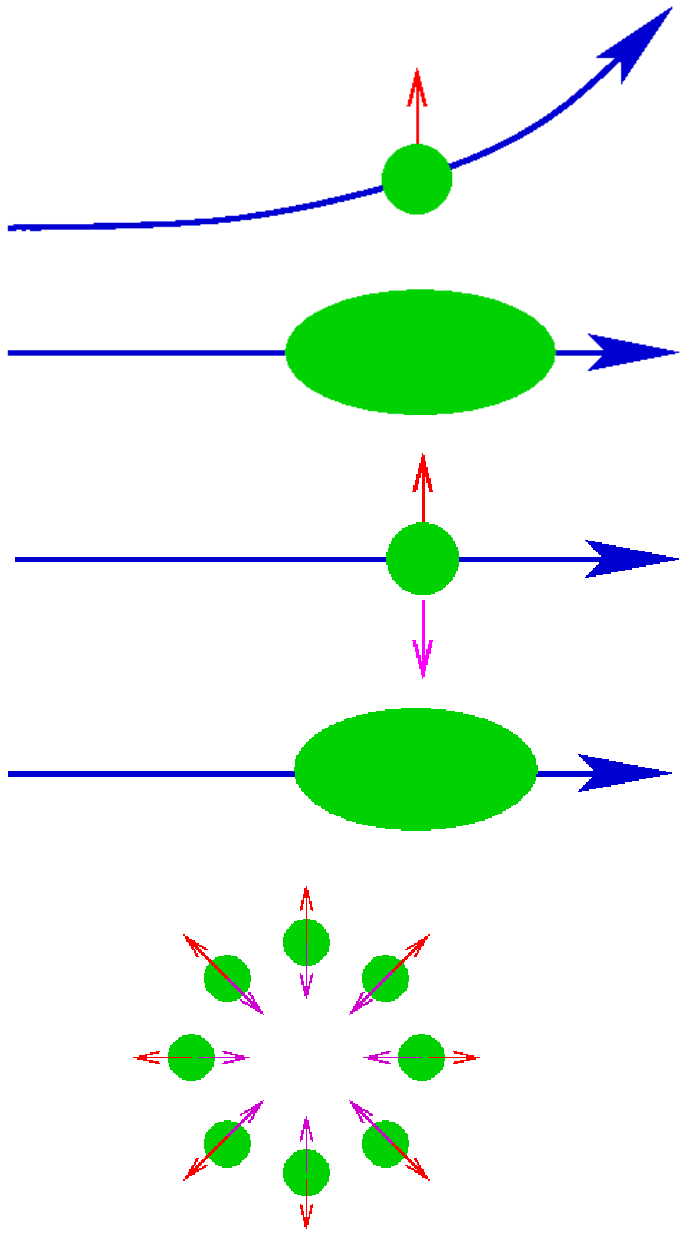
- Hence the potential

$$\phi(x, z) = V_0 \mathbb{R}(\frac{z + jx}{d})^{4/3} \qquad z = \rho \cos\theta \qquad x = \rho \sin\theta$$



- Hence for the source electrode: $4\theta / 3 = \pi / 2$
- And for the extraction electrode: $(\rho / d)^{4/3} \cos(4\theta / 3) = 1$

... and then?



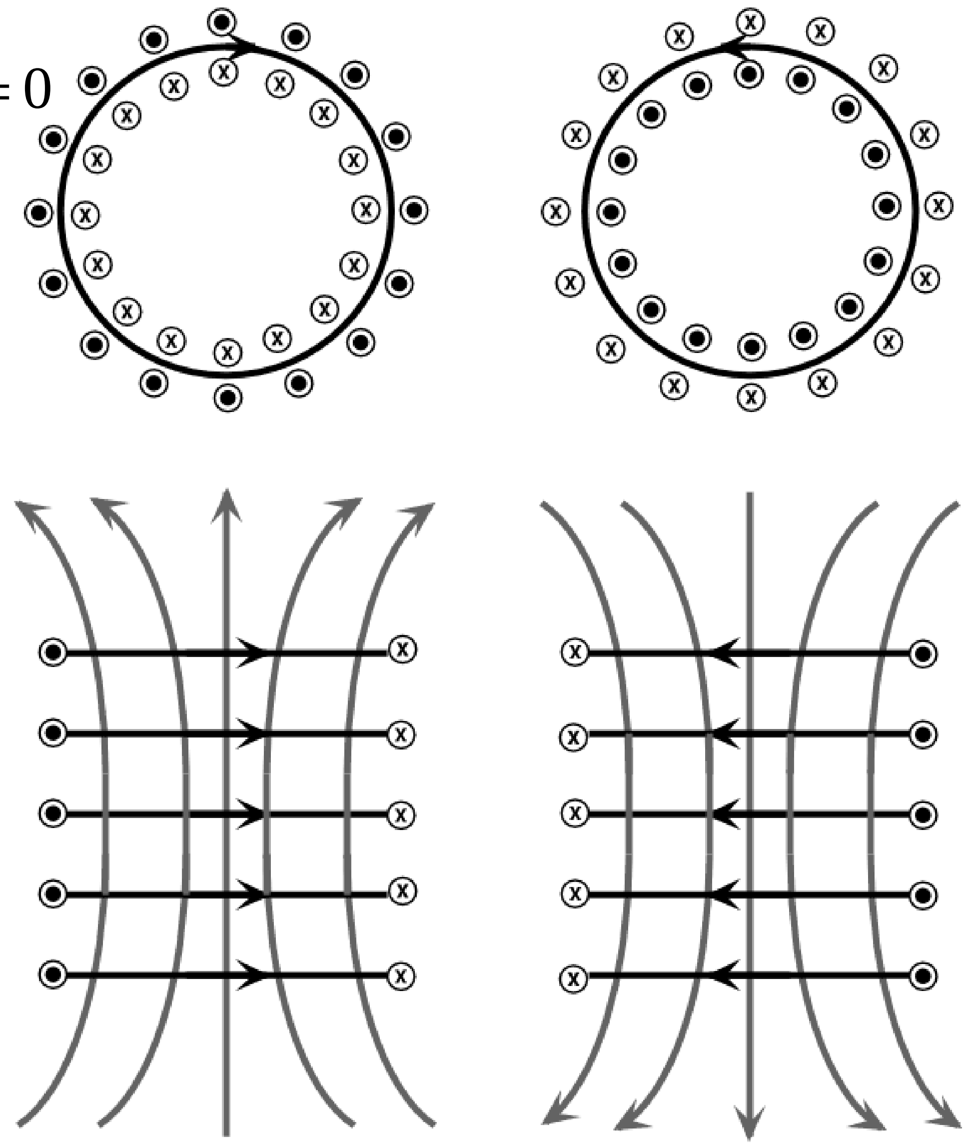
- After the anode space charge effects are still present.
- It is not possible to use an electrostatic solution anymore.
- The compensating field must have a circular symmetry...
- This is not easy to achieve!

Compensating solenoid (1)

- The solution is to use a compensating solenoid.

$$\vec{F} = qv\vec{z} \wedge B\vec{z} = 0$$

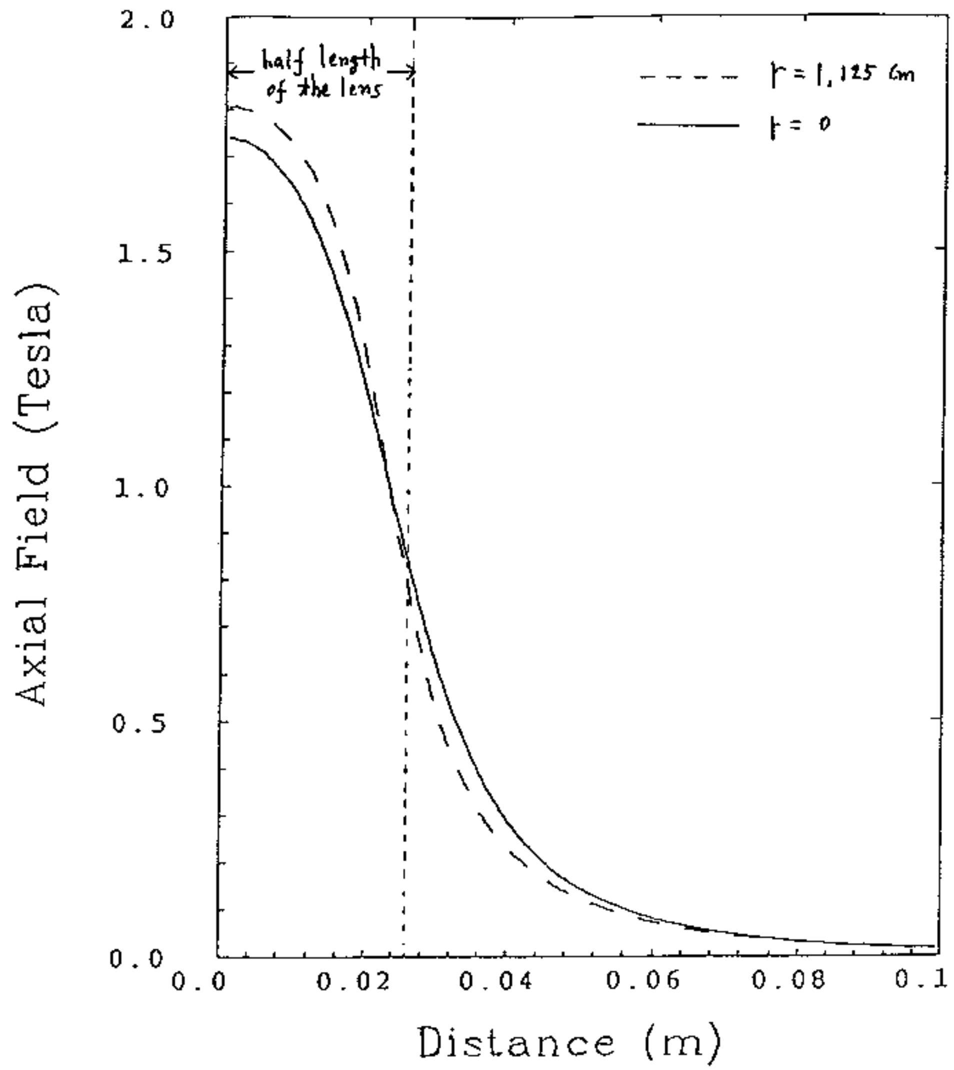
- Inside the solenoid the field is so that the angular momentum of the particles couples with the field.
- BUT at the edge of the solenoid the particles decouple from the field and receive a transverse kick (Busch theorem).



Busch theorem

- Canonical angular momentum must be conserved.
- In a solenoid charged particles couple their transverse momentum to the field.
- At the edge of the solenoid the field suddenly decreases.
- To conserve the correct coupling the particles will be deflected toward more intense field (ie the middle of the solenoid)
- This will induce a focussing effect.

$$rP_{\phi} + \frac{q}{2\pi}\oint B = 0$$

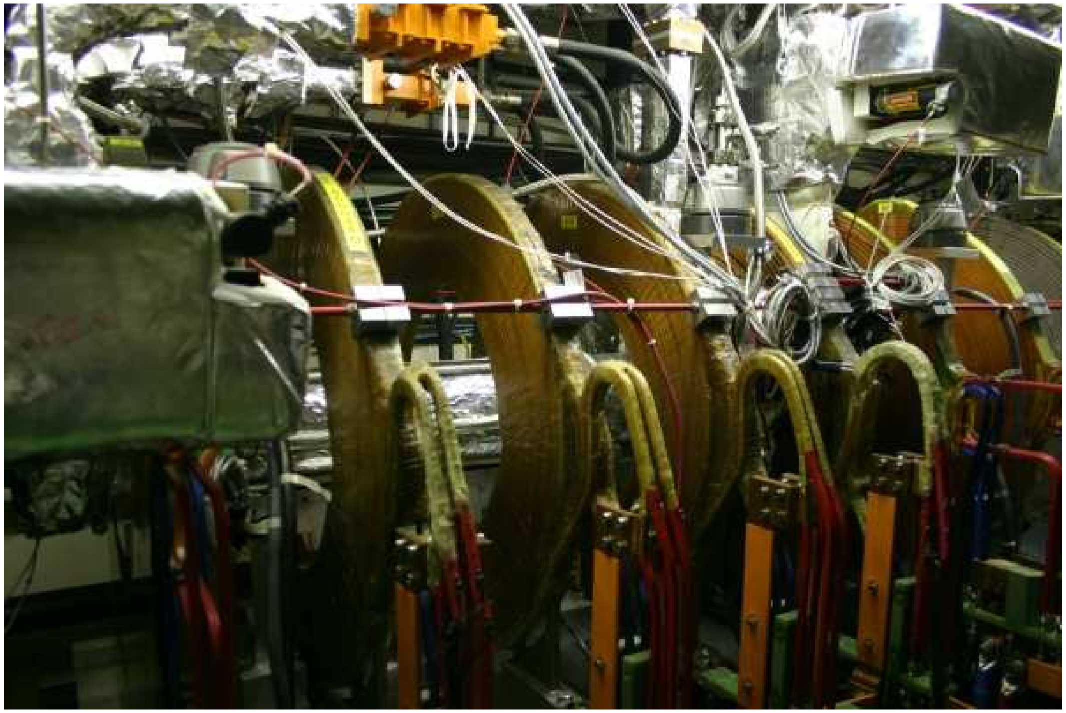


Compensating solenoid (2)

- To achieve the best compensation effect, several small solenoids are much better than a big one (as this maximizes the edge effect).
- This can be seen on this picture of the Diamond gun.
- Once high energies are reached the particles travel fast enough so that the space charge do not need to be compensated any more.

Quiz

- On this image of the CLIC injector the electrons travel from the left to the right.
- In which direction does the current flow in the solenoid to compensate the space-charge effect?



a) Clockwise

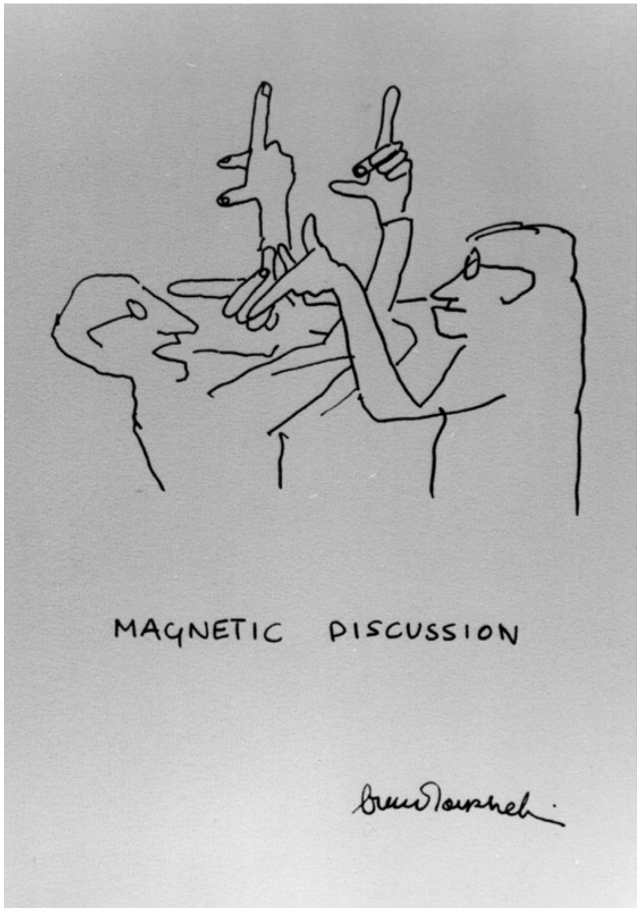
b) Counter clockwise

c) It does not matter

$$rP_{\phi} + \frac{q}{2\pi}\oint B = 0$$

Answer: a

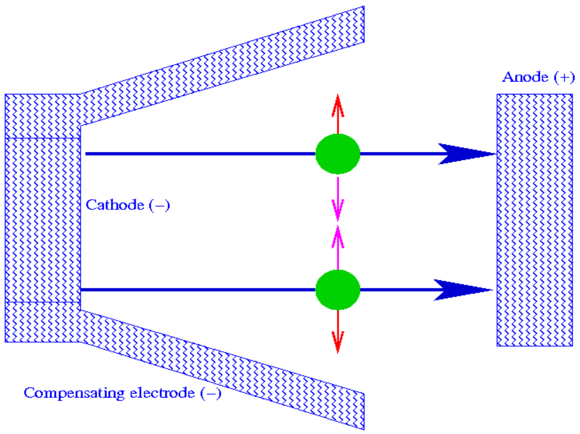
- Busch theorem: $rP_\phi + \frac{q}{2\pi}\oint B = 0$
- To get a negative transverse kick the second term must be positive.
- q is negative. $rP_\phi + \frac{-e}{2\pi}\oint B = 0$
- So the flux must be positive.
- The electrons must travel in the direction of the flux.
- The current must circulate in a clockwise direction.

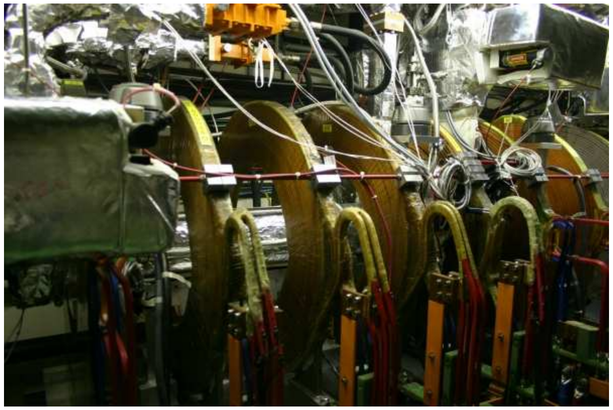


Space charge summary

- We have seen that space charge can be compensated either:
  - by having a compensating electrode
  - by using a solenoid.

- Now we will study why it is important to have a small beam size from the beginning.



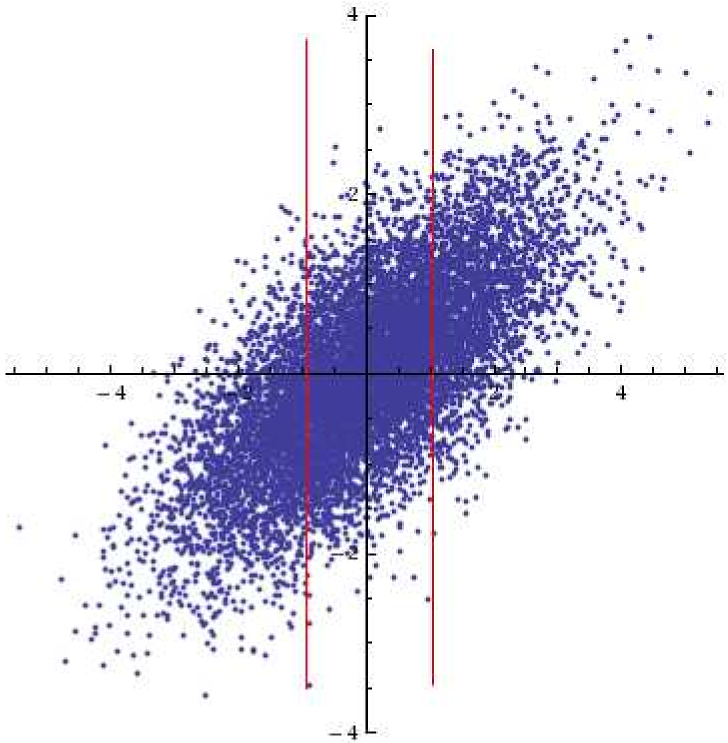


Beam size and magnet aperture

- One of the motivations for keeping the beam small is that magnets have a limited aperture.
- The larger the aperture the more difficult it is to keep a uniform field
  (and the bigger the magnet is).
- It is recommended to have a beam pipe 5 times larger than the RMS size of the beam.

Clipping

- If the beam pipe is too small some particles will be clipped.
- With a 1 sigma aperture most of the particles would be lost.
- Do not forget that the beam is gaussian: whatever the aperture, some particles will always be clipped...
- The aperture of the beam must be chosen so that clipping is limited.



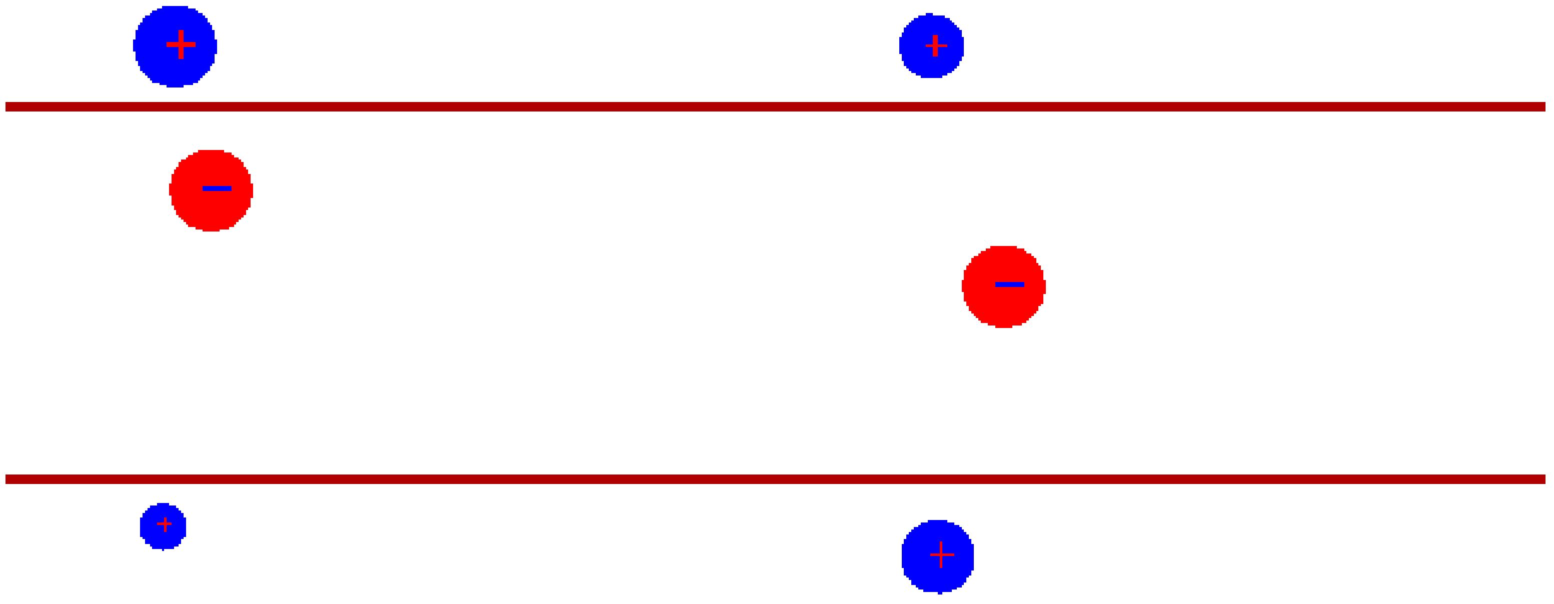
Wakefield issues

- Electrons produce an electromagnetic wave behind them.
- This can be compared the to wake of a boat and is called wakefield.
- Imagine what would happen if there was a second surfer on the picture below...
- How good is the wake for the walls of the canal?
  It is not good for the beam pipe either!

Source: Reuters

Impedance issues

- Charged particles travelling near a conductor induce image charges (induced current).
- This image current dissipate power in the beam pipe.
- The smaller the beam pipe, the higher the induced charge and thus the highest the losses.
- The impedance of a beam pipe must be carefully controlled!



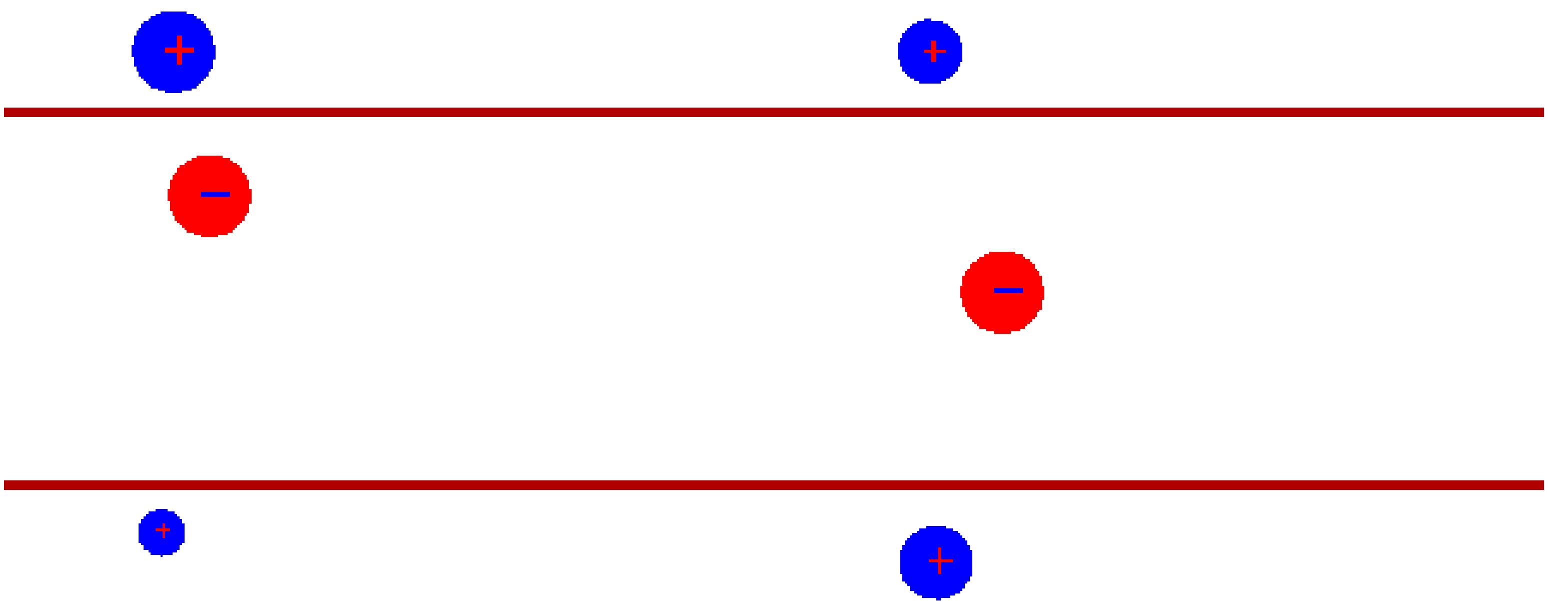
Quiz

- In the example below, which particle will create the more induced current on the beam pipe?

a) The particle on the left hand-side

b) The particle on the right hand-side

c) Both will induce the same current

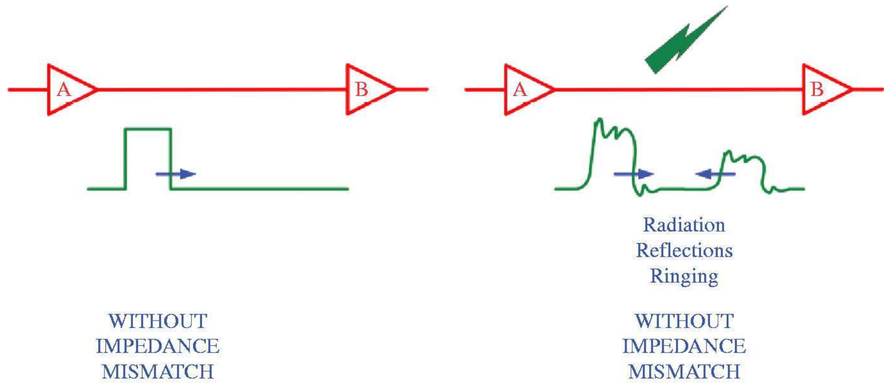


Answer (c)

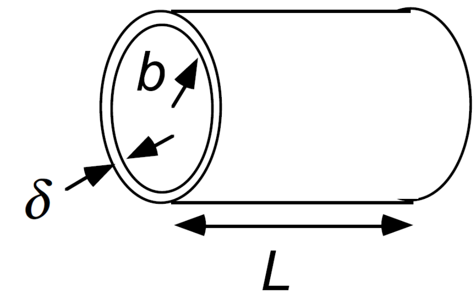
- Apply Faraday's law: $$\mathcal{E} = -\frac{d\Phi_B}{dt} \ ,$$
- The total flux going through the beam pipe is the same for both particles.
- Hence the total current induced on the beam pipe by both particles will be the same.
- ***Remember this result, we will use it Tomorrow!***

Impedance matching

- In a RF circuit an impedance mismatch will result in a reduced transmission at the interface.
- The same is true in an accelerator: an impedance mismatch is likely to induce a reflective wave at the interface.
- This will induce a loss of power and an emittance increase.
- In a synchrotron the impedance of all beam pipe elements is carefully controlled.
- This is less important in a transfer line where the beam passes only once.



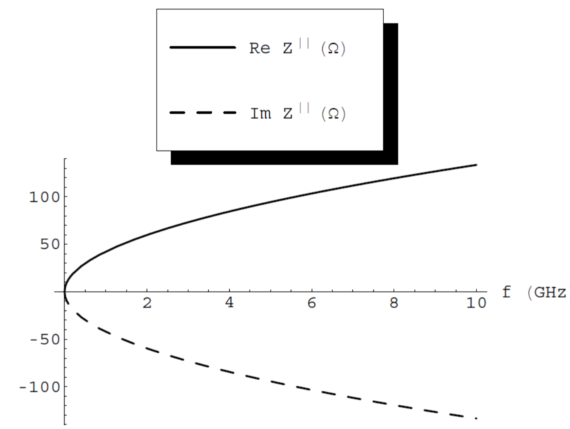
Impedance



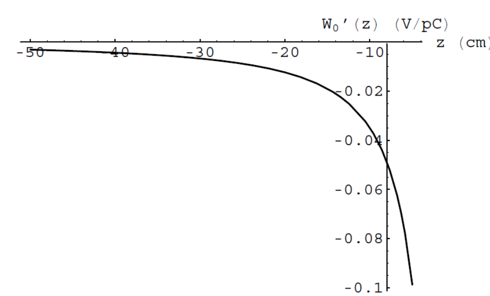
$$\delta = \sqrt{\frac{2}{\sigma\mu\omega}}$$ (skin dep

$$\frac{R_{wall}(\omega)}{L} = \frac{1}{\sigma A} = \frac{1}{\sigma 2\pi b\delta} = \frac{1}{\sigma 2\pi b\delta} = \frac{1}{2\pi b}\sqrt{\frac{\mu\omega}{2\sigma}}$$

- Let’s consider a section of beam pipe.
- The impedance depends on the skin depth and the area of the pipe.
- C=ring circumference
- This impedance will induce a wake field, intense behind the beam but with a long tail => long distance effect for the following bunch.

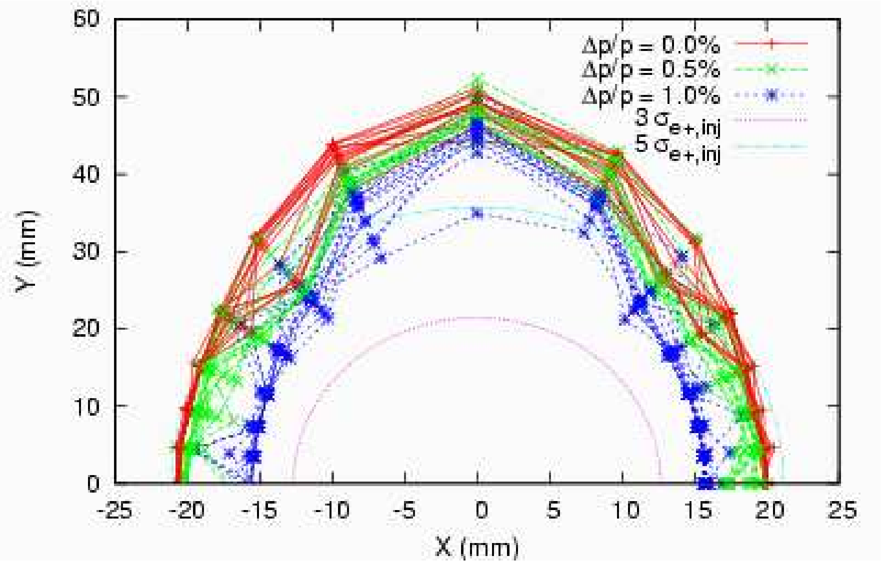


$$W_0'(z) = -C\frac{c}{4\pi b}\sqrt{\frac{c\mu}{\pi\sigma}}\frac{1}{\sqrt{|z|^3}}$$



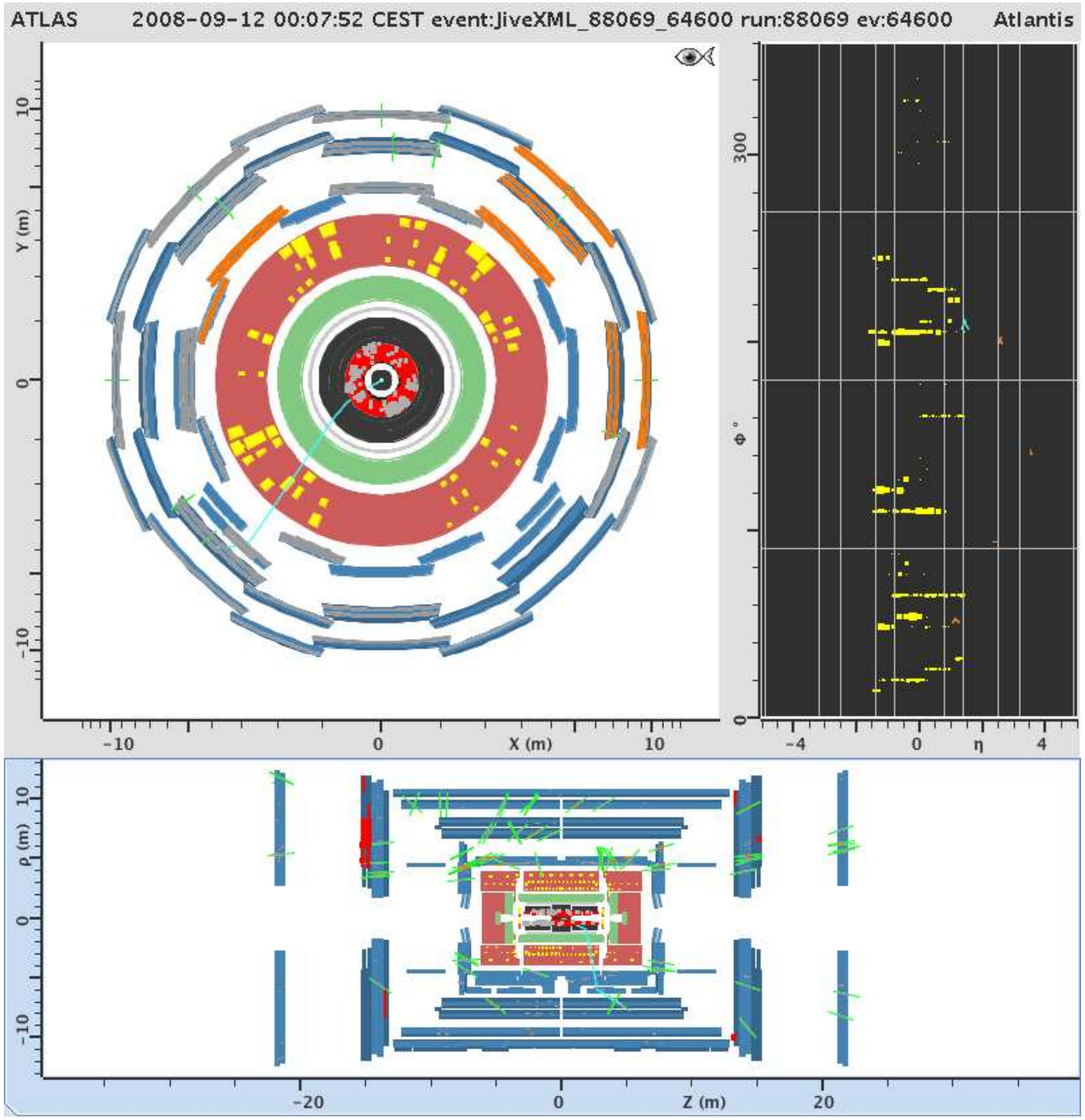
Magnet aperture

- Clipping, wakefield and impedances issues lead to poor accelerator performances.
- To avoid these effects the trajectory of the beam in the accelerator must be simulated.
- Tracking software are used to do this: they study how particles move from one location to the next.
- Instead of tracking each particle individually it is enough to track the envelope of the beam.
- We will now study effects that lead to particle loss and lifetime reduction.

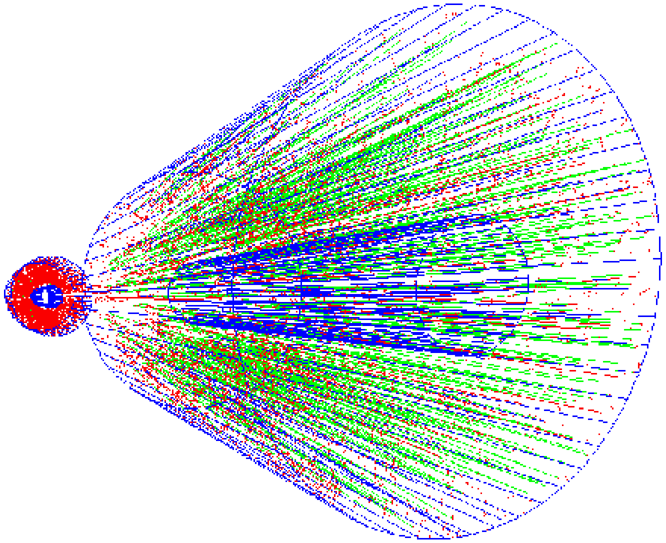


Halo

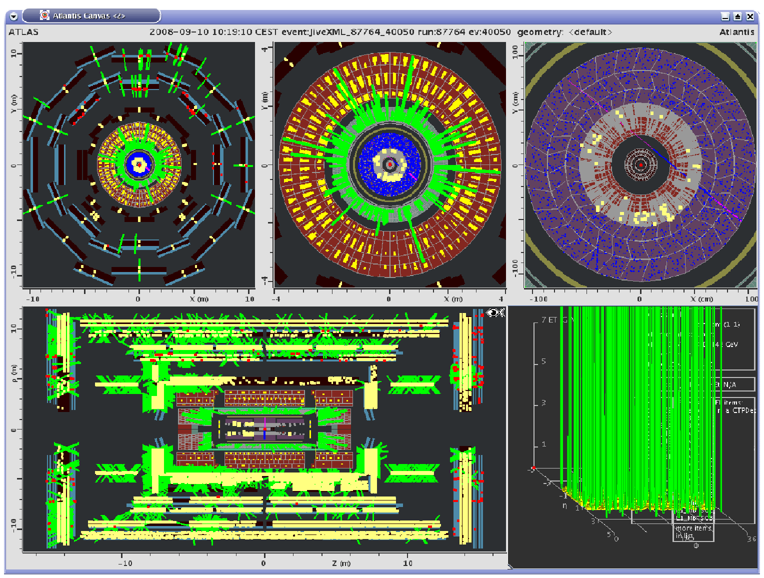
- Charged particles attract particles with the opposite charge.
- This creates a halo of particles with opposite charge.
- This halo of particles is not confined to the beam and thus it can damage equipment outside the beam pipe also.
- It is a source of beam instabilities and can lead to significant emittance growth.



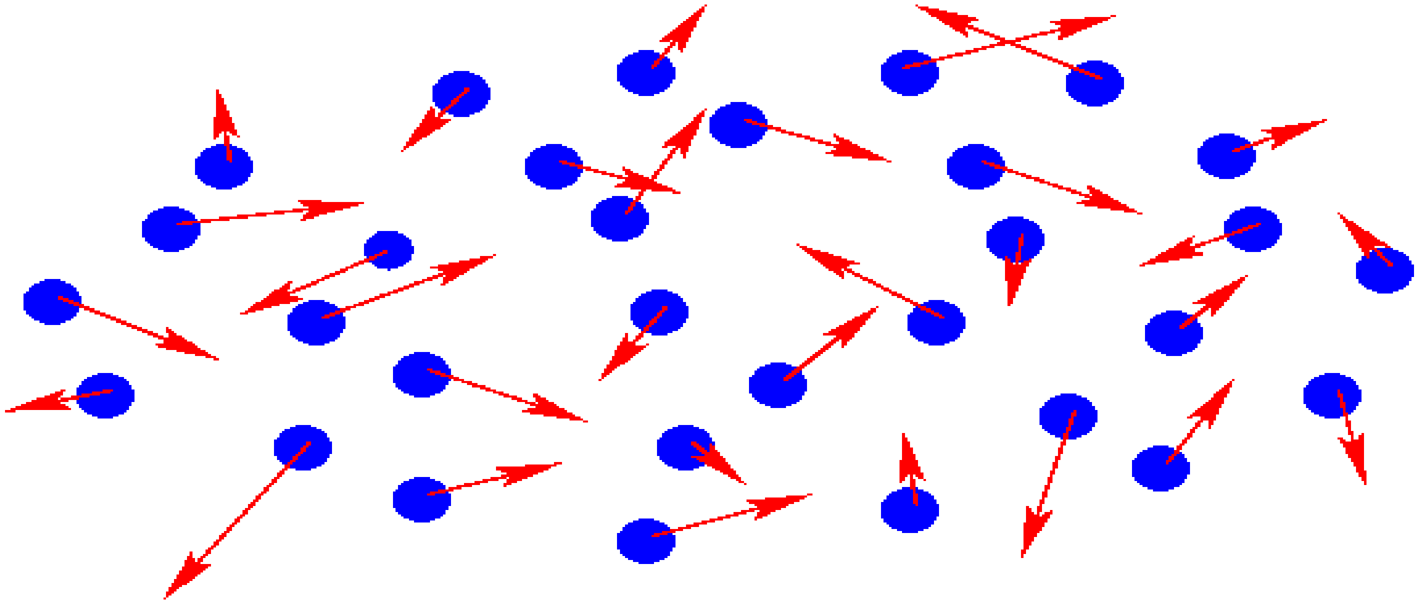
Beam gas collisions



- Even in ultra high vacuum their are some gas atoms remaining in the beam pipe.
- When a high energy particle hits one of these atoms it is smashes it and creates a shower.
- Repeated beam-gas event lead to significant losses of particles in the accelerator.



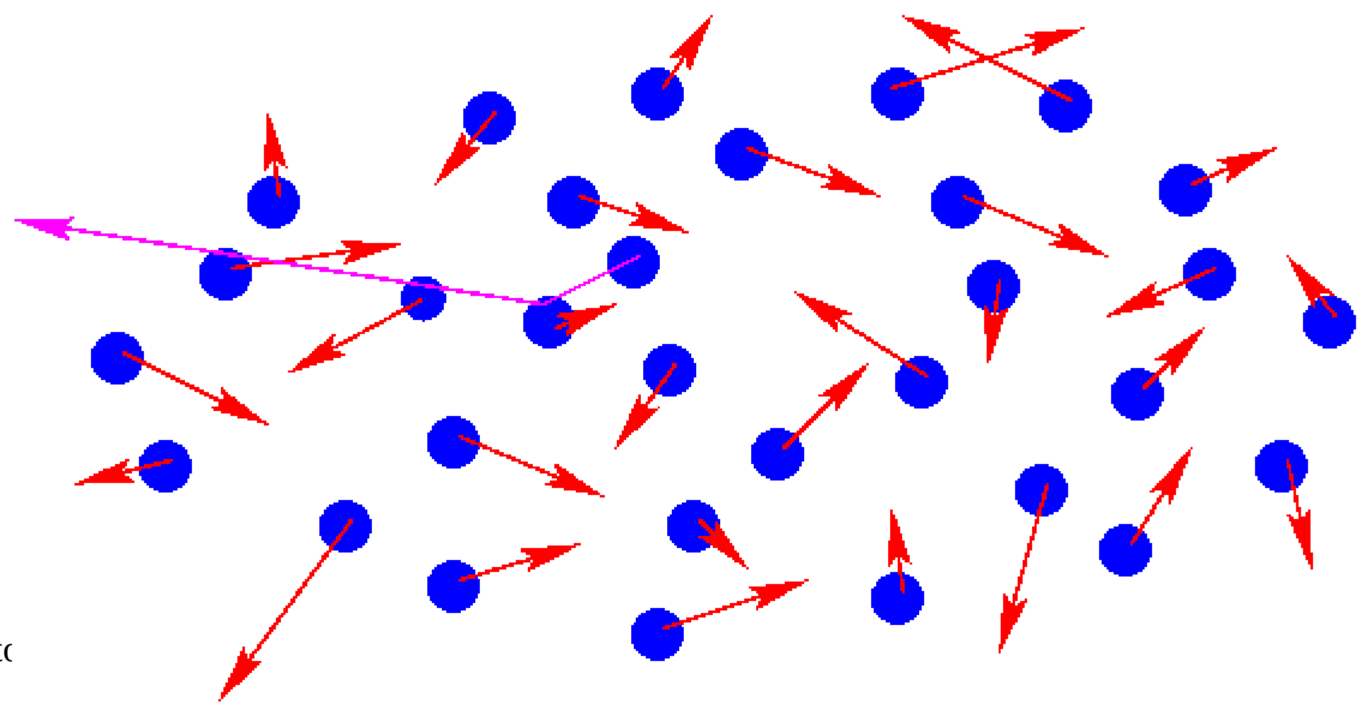
Intra-beam scattering



- We have seen that inside the beam the particles behave somewhat like a gas.
- Coulomb collisions do occur between the particles.
- These collisions lead to a momentum transfer between the particles and thus an emittance coupling.
- Beam with a larger emittance will experience more IBS.

Touschek effect

- In addition to Coulomb scattering, hard scattering can also occur.
- In most cases this will lead to one particle being pushed out of the acceptable beam orbit and thus being lost soon after.
- Touschek scattering occurs in high current beams.



Touschek lifetime

- Time needed for the beam to loose half its charge due to the Touschek effect

$$\frac{1}{\tau} = \frac{r_e^2 c q}{8\pi e \gamma^3 \sigma_s} \cdot \frac{1}{C} \cdot \oint_C \frac{F\left(\left(\delta_{acc}(s) \big/ \gamma \sigma_{x'}(s)\right)^2\right)}{\sigma_x(s)\sigma_z(s)\sigma_{x'}(s)\delta_{acc}^2(s)} ds \qquad (1)$$

with

r_e the classical electron radius,

q the bunch charge,

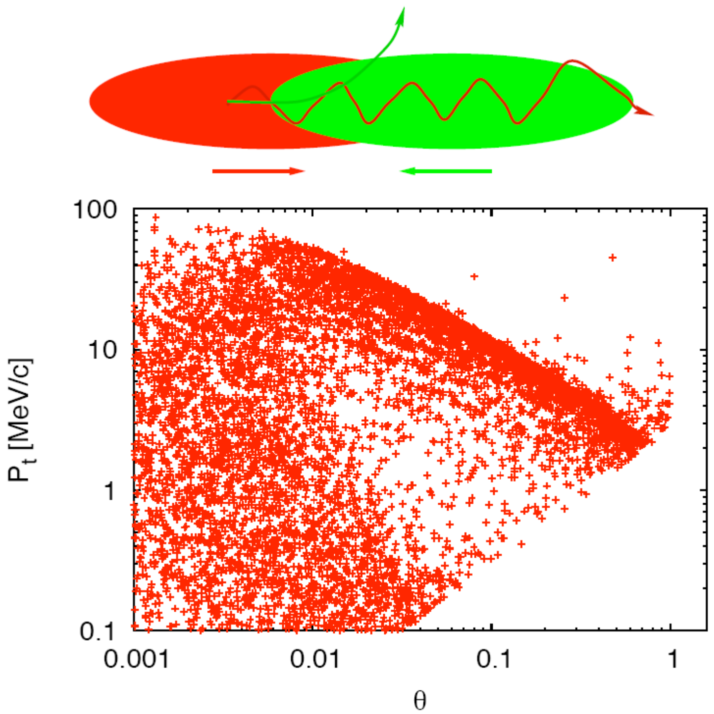
σ_s the rms bunch length, assumed to be constant along the lattice (usually valid for storage rings),

C the machine circumference,

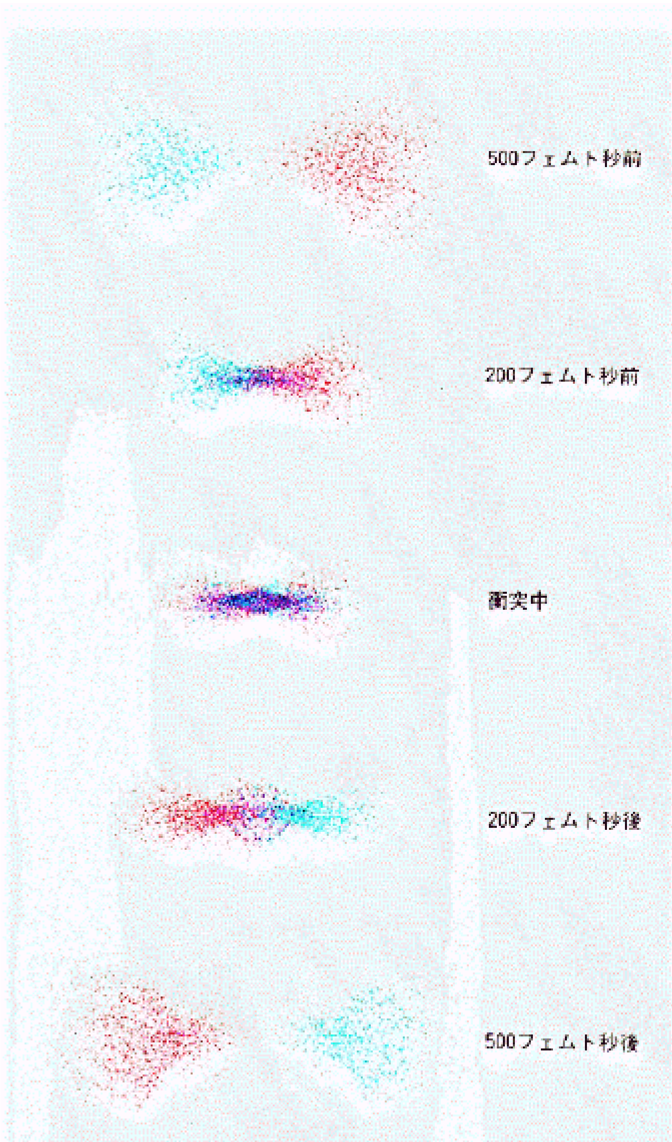
$\delta_{acc}(s)$ the local relative energy acceptance, which can be determined by the RF-system or by the lattice acceptance, and

Beam beam effects (1)

- In a collider the two beams feel each other's electric field well before and well after colliding.
- Given that the particles come very close to each other, this lead to very intense forces.
- These forces lead to significant disruption of the beam.



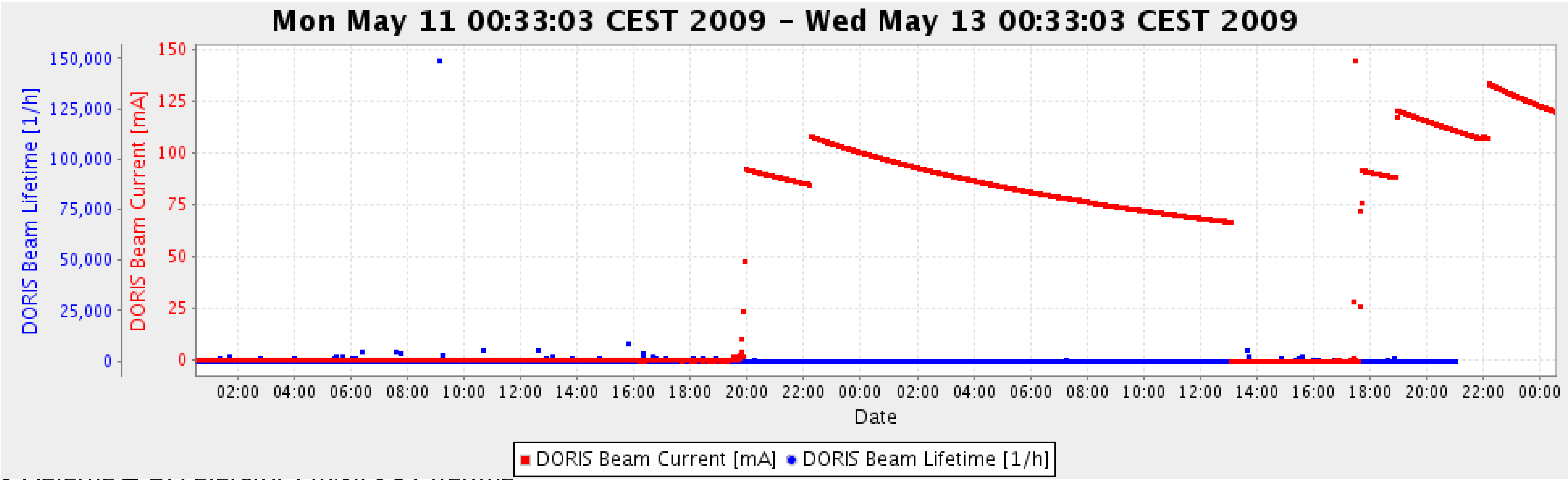
Beam beam effect (2)



- At the interaction point the two beams self-focus onto each other.
- If the self focussing is too strong this can lead to a large emittance growth.
- If the two beam are not perfectly aligned this will also lead to large transverse deflection.

Beam lifetime (1)

- Each of these effects leads to the loss of particles.
- The number of particles will decays slowly until it reaches 0.
- This is called the beam lifetime.

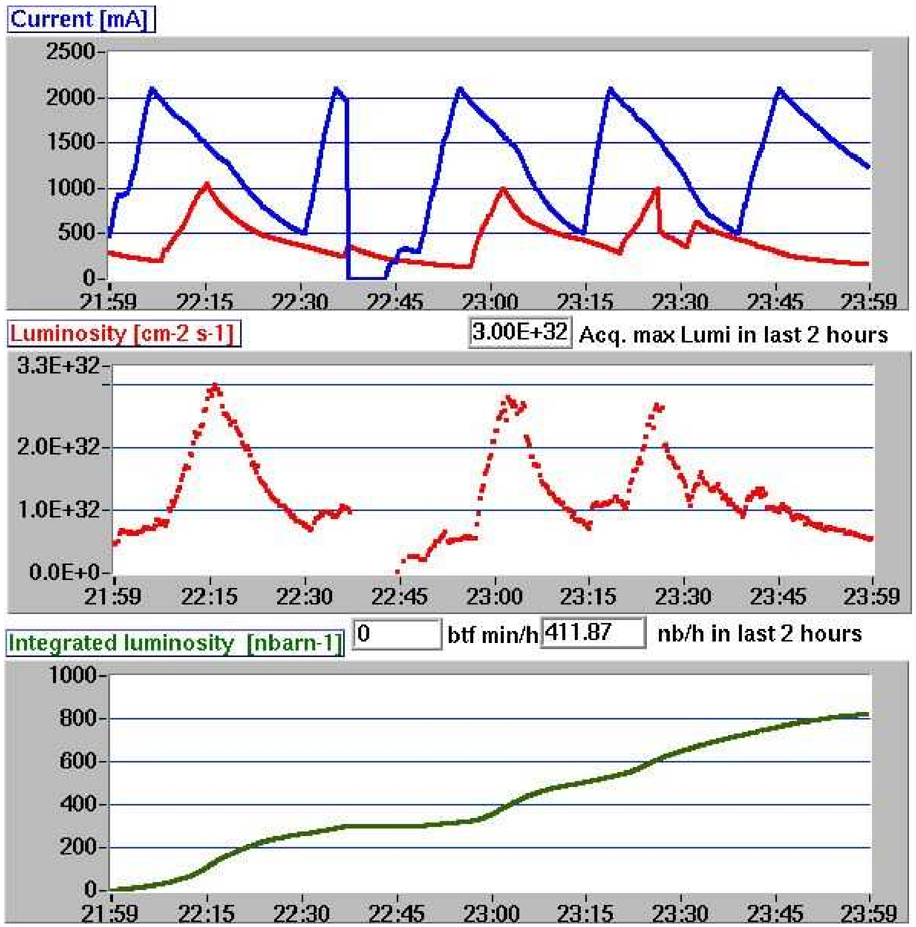


Beam lifetime (2)

- The total lifetime is the sum of the lifetime for each effect.

$$\frac{1}{\tau_{beam}} = \frac{1}{\tau_{gas}} + \frac{1}{\tau_{IBS}} + \frac{1}{\tau_{Touschek}} + \frac{1}{\tau_{bb}}$$

- Maximising the lifetime of the accelerator allows more stable and reliable operations.



Summary

- Space charge is a significant source of emittance growth. To ensure reliable operations it must be compensated.
- The size of the the beam must be checked carefully. A too small beam is a source of instabilities that can significantly reduce the accelerator performances.
- Inside the beam there are phenomena that also can lead to reduced performances and particle losses. The parameters of the beam must be chosen so that these effects are minimised.
- Tomorrow we will study how to measure the properties of the beam.